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European Journal of Mechanics B/Fluids 26 (2007) 105–131

Half-space problem of the nonlinear Boltzmann equation for weak evaporation and condensation of a binary mixture of vapors

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Received 9 January 2006; received in revised form 12 April 2006; accepted 13 April 2006
Available online 5 June 2006

Abstract

The half-space problem of evaporation and condensation of a binary mixture of vapors is investigated on the basis of the kinetic theory of gases. Assuming the Mach number of the normal component of the flow is small, a solution of the Boltzmann equation that varies slowly in the scale of the molecular mean-free-path (slowly varying solution) is introduced. Then a fluid-dynamic system that describes the behavior of the slowly varying solution is derived by a systematic asymptotic analysis. The analytical expression of the conditions allowing steady evaporation or condensation is derived from that system. We analyze the qualitative difference between the conditions in the evaporation and condensation cases: four conditions are needed in the former case while only one condition is required in the latter case. The present paper extends a earlier contribution of the first author for the BGK-type model equation [S. Takata, Half-space problem of weak evaporation and condensation of a binary mixture of vapors, in: Capitelli M. (Ed.), Rarefied Gas Dynamics, AIP, New York, 2005, pp. 503–508] to the Boltzmann equation. The extension is achieved by considering the linear stability of the far field in the case of evaporation and the H theorem, the monotonic decrease of the flux of Boltzmann's H function, in the case of condensation.

PACS: 51.10.+y; 47.45.-n; 47.70.Nd; 05.20.Dd

Keywords: Evaporation; Condensation; Binary mixture; Boltzmann equation; Kinetic theory of gases; Half-space; Slowly varying solution; Boundary with suction; H theorem

1. Introduction

Consider a vapor occupying a half-space bounded by a planar surface of its condensed phase. The vapor is supposed to be in a uniform equilibrium state at a far distance with flowing from (evaporation) or onto (condensation) the surface. Investigation of the steady behavior of the vapor, which we call the half-space problem of evaporation and condensation, is one of the most fundamental boundary-value problems of the Boltzmann equation and has been intensively studied (see, for example, [1–5] and the references therein). The problem has a practical importance

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0997-7546/$ – see front matter
doi:10.1016/j.euromechflu.2006.04.003

because it provides the basic equation in the continuum gas dynamics (the Euler set of equations) with the boundary conditions at the surface of the condensed phase [6]. In a half-space, evaporation or condensation can take place only when certain relations among parameters are satisfied, i.e., the problem is solvable conditionally. It is these relations that are used as the boundary conditions for the Euler set.

One of the interesting features of the problem is that the solution has a qualitatively different structure between evaporation and condensation. The mechanism of the difference was clarified by Sone in [7], where the problem is studied by weakly nonlinear analysis by assuming a flow of small Mach number. In the half-space problem, studied is a transition region from the surface to the region in a uniform equilibrium state (the outer Euler region). The transition region typically has a thickness of a few mean free paths of a molecule and may be considered as the so-called Knudsen layer. However, it is often observed that this region becomes much thicker than the mean free path when condensing with a small Mach number. In [7], assuming the Mach number of the flow is small, it is clarified that (i) in the case of condensation, the transition region is subdivided into the Knudsen layer adjacent to the surface and a region described by the Navier–Stokes set that connects the Knudsen layer with the outer Euler region; (ii) in the case of evaporation, there is no subdivision and the Knudsen layer directly connects the surface with the outer Euler region. Corresponding to the structural difference, the relations among parameters when evaporating are qualitatively different from those when condensing. Incidentally, the structure of the transition region in the case of condensation is closely related to the so-called suction boundary layer in the conventional gas dynamics (e.g., [8]). Although the feature described here was clarified by assuming a single-species vapor, it is natural to expect that it would essentially remain unchanged when the vapor is composed of multiple species.

In the present paper, we will study the half-space problem of evaporation and condensation for a binary mixture of vapors when the Mach number of the normal component of the flow is small. Our aim is to show that essentially the same structural difference of the transition region between evaporation and condensation arises and to provide the conditions that allow the evaporation and condensation explicitly. As will be clear later, following Sone [7], we introduce a solution of the Boltzmann equation, which we call the *slowly varying solution* [1], with the length scale of variation of the mean free path divided by the Mach number, derive the fluid-dynamic system describing its behavior, and discuss the properties of the solution of the system. The difference of the properties between evaporation and condensation cases induces the structural difference of the transition region.

Preceding the present work, the first author studied the same problem by the use of the BGK-type model Boltzmann equation [9–11] in the case where there is no flow in the direction tangential to the surface [12]. In this case the fluid-dynamic system can be (formally) directly solved to show the monotonic behavior of the macroscopic quantities. Such monotonic property is also true in the case of a single-species vapor for the Boltzmann equation for general molecular models; and it was fully used in Refs. [12,7,1] in deriving the conclusion. However, the macroscopic quantities do not monotonically vary in general in the case of mixtures. It is the objective of the present paper to generalize the previous work [12] to the case of the Boltzmann equation for general intermolecular potentials. This objective is achieved by taking somewhat indirect way: the linear stability analysis in the case of evaporation and the use of the H theorem, the monotonic decrease of the flux of Boltzmann's H function, in the case of condensation.

2. Problem

We consider a binary mixture of vapors, say species A and B, in a half-space in contact with their condensed phase. The condensed phase is kept at a uniform temperature T_w, and its interface with the vapors is located at $X_1 = 0$, where X_i is the rectangular coordinate system. The mixture of vapors occupies the region of $X_1 > 0$ and is uniform at a far distance from the interface with pressure p_∞, temperature T_∞, partial pressure p_∞^A of species A, and flow velocity $\boldsymbol{v}_\infty = (v_{1\infty}, v_{2\infty}, 0)$. The flow speed in the X_1-direction at a far distance, $|v_{1\infty}|$, is supposed to be small compared to the sound speed. We will investigate the steady behavior of the vapors on the basis of the kinetic theory of gases. In the analysis, we assume that (i) the behavior of the vapors is described by the Boltzmann equation for gaseous mixtures and that (ii) the velocity distribution of the molecules of species α ($\alpha = \mathrm{A}, \mathrm{B}$) incoming from the interface is the half-Maxwellian at rest which is characterized by the temperature T_w and a pressure p_w^α. Here p_w^α is the partial pressure of species α in the mixture saturated in contact with the condensed phase at temperature T_w. Physically, p_w^α depends not only on the temperature but also on the constituents and the constituent ratio of the condensed phases. In the present paper, we specify not these two quantities but instead p_w^α and T_w independently one another as certain constants. Therefore, we implicitly assume that the change of the composition of the condensed phase in the process of evaporation/condensation is so small that it can be neglected. Hereinafter, just for brevity, we call p_{w}^{α} the “saturation pressure” of species α at the interface and $p_{\mathrm{w}} \equiv p_{\mathrm{w}}^{\mathrm{A}} + p_{\mathrm{w}}^{\mathrm{B}}$ the total “saturation pressure” at the interface, respectively. Incidentally, the assumption (ii) implies the perfect accommodation of the molecules coming to the interface from the gas phase.

3. Formulation of the problem

3.1. Basic equation and boundary condition

Let us denote by $\boldsymbol{\xi}$ (or ξ_i) the molecular velocity and by $F^{\alpha}(X_1, \boldsymbol{\xi})$ the velocity distribution function of the molecules of species α ($\alpha = \mathrm{A}, \mathrm{B}$). In the sequel the Greek letters α, β, and γ are symbolically used to represent the species of vapors, i.e., $\{\alpha, \beta, \gamma\} = \{\mathrm{A}, \mathrm{B}\}$.

The steady and spatially one-dimensional Boltzmann equation for a binary gaseous mixture is written as

$$\xi_1 \frac{\partial F^{\alpha}}{\partial X_1} = \sum_{\beta=\mathrm{A},\mathrm{B}} J^{\beta\alpha}\big(F^{\beta}, F^{\alpha}\big), \tag{1}$$

with

$$J^{\beta\alpha}(F, G) = \int \big(F'_{*}G' - F_{*}G\big) B^{\beta\alpha}\big(|\boldsymbol{e}\cdot\boldsymbol{V}|/V, V\big)\,\mathrm{d}\Omega\big(\boldsymbol{e}\big)\,\mathrm{d}\boldsymbol{\xi}_{*}, \tag{2}$$

where

$$F'_{*} = F(X_1, \boldsymbol{\xi}'_{*}), \quad G' = G(X_1, \boldsymbol{\xi}'), \quad F_{*} = F(X_1, \boldsymbol{\xi}_{*}), \quad G = G(X_1, \boldsymbol{\xi}),$$
$$\boldsymbol{\xi}' = \boldsymbol{\xi} + \frac{\mu^{\beta\alpha}}{m^{\alpha}}(\boldsymbol{e}\cdot\boldsymbol{V})\boldsymbol{e}, \qquad \boldsymbol{\xi}'_{*} = \boldsymbol{\xi}_{*} - \frac{\mu^{\beta\alpha}}{m^{\beta}}(\boldsymbol{e}\cdot\boldsymbol{V})\boldsymbol{e},$$
$$\mu^{\beta\alpha} = \frac{2m^{\alpha}m^{\beta}}{m^{\alpha}+m^{\beta}}, \quad \boldsymbol{V} = \boldsymbol{\xi}_{*} - \boldsymbol{\xi}, \quad V = |\boldsymbol{V}|,$$

and $\boldsymbol{e}$ is a unit vector, $\mathrm{d}\Omega(\boldsymbol{e})$ is the solid-angle element in the direction of $\boldsymbol{e}$, $\mathrm{d}\boldsymbol{\xi}_{*} = \mathrm{d}\xi_{*1}\,\mathrm{d}\xi_{*2}\,\mathrm{d}\xi_{*3}$, m^{α} is the mass of a molecule of species α, and $B^{\beta\alpha}$ is a nonnegative function of its arguments whose functional form is determined by the intermolecular potential between a molecule of species β and that of species α. The integration in (2) is carried out over the whole space of $\boldsymbol{\xi}_{*}$ and over the unit sphere.

The boundary conditions for species α at the interface ($X_1 = 0$) and at infinity ($X_1 \to \infty$) are written as

$$F^{\alpha} = \frac{p_{\mathrm{w}}^{\alpha}/kT_{\mathrm{w}}}{(2\pi kT_{\mathrm{w}}/m^{\alpha})^{3/2}} \exp\bigg(-\frac{m^{\alpha}|\boldsymbol{\xi}|^2}{2kT_{\mathrm{w}}}\bigg) \quad \text{for } \xi_1 > 0 \text{ at } X_1 = 0, \tag{3a}$$

$$F^{\alpha} = \frac{p_{\infty}^{\alpha}/kT_{\infty}}{(2\pi kT_{\infty}/m^{\alpha})^{3/2}} \exp\bigg(-\frac{m^{\alpha}|\boldsymbol{\xi}-\boldsymbol{v}_{\infty}|^2}{2kT_{\infty}}\bigg) \quad \text{as } X_1 \to \infty, \tag{3b}$$

where k is the Boltzmann constant and $p_{\infty}^{\mathrm{B}} = p_{\infty} - p_{\infty}^{\mathrm{A}}$. Eqs. (1)–(3) is the boundary-value problem to be studied in the present paper.

Before proceeding further, here we introduce macroscopic quantities for the later convenience. The molecular number density n^{α}, the mass density ρ^{α}, the flow velocity $\boldsymbol{v}^{\alpha} = (v_1^{\alpha}, v_2^{\alpha}, 0)$, the partial pressure p^{α}, and temperature T^{α} of species α are defined as

$$n^{\alpha} = \int F^{\alpha}\,\mathrm{d}\boldsymbol{\xi}, \qquad \rho^{\alpha} = m^{\alpha}n^{\alpha}, \tag{4a}$$

$$v_i^{\alpha} = \frac{1}{n^{\alpha}}\int \xi_i F^{\alpha}\,\mathrm{d}\boldsymbol{\xi} \quad (i = 1, 2), \tag{4b}$$

$$p^{\alpha} = n^{\alpha}kT^{\alpha} = \frac{1}{3}\int \big|\boldsymbol{\xi} - \boldsymbol{v}^{\alpha}\big|^2 m^{\alpha}F^{\alpha}\,\mathrm{d}\boldsymbol{\xi}, \tag{4c}$$

and their counterparts of the mixture, i.e., the molecular number density n, the mass density ρ, the flow velocity $\boldsymbol{v} = (v_1, v_2, 0)$ based on the momentum flow, the pressure p, and the temperature T, are defined as

$$n = n^{\mathrm{A}} + n^{\mathrm{B}}, \qquad \rho = \rho^{\mathrm{A}} + \rho^{\mathrm{B}}, \tag{5a}$$

$$v_i = \frac{1}{\rho}\big(\rho^{\mathrm{A}} v_i^{\mathrm{A}} + \rho^{\mathrm{B}} v_i^{\mathrm{B}}\big) \quad (i = 1, 2), \tag{5b}$$

$$p = nkT = \sum_{\alpha=\mathrm{A},\mathrm{B}} \left(p^\alpha + \frac{1}{3}\rho^\alpha |\boldsymbol{v}^\alpha - \boldsymbol{v}|^2 \right). \tag{5c}$$

Note that Dalton's law does not hold in general in our definition. Also note that the X_3 component of the flow velocity is assumed to vanish from the beginning, because one can seek the solution of (1)–(3) as an even function of ξ_3. Because of the same reason, the X_2X_3 component of the stress tensor and the X_3 component of the heat-flow vector may be assumed to vanish. The other components of the stress tensor p_{ij}^α and the heat flow q_i^α of species α are defined as

$$p_{ij}^\alpha = \int \big(\xi_i - v_i^\alpha\big)\big(\xi_j - v_j^\alpha\big) m^\alpha F^\alpha \, \mathrm{d}\boldsymbol{\xi} \quad (i, j = 1, 2), \tag{6a}$$

$$p_{33}^\alpha = \int \xi_3^2 m^\alpha F^\alpha \, \mathrm{d}\boldsymbol{\xi}, \tag{6b}$$

$$q_i^\alpha = \frac{1}{2} \int \big(\xi_i - v_i^\alpha\big) \big|\boldsymbol{\xi} - \boldsymbol{v}^\alpha\big|^2 m^\alpha F^\alpha \, \mathrm{d}\boldsymbol{\xi} \quad (i = 1, 2), \tag{6c}$$

and the counterparts of the mixture, p_{ij} and q_i, are as

$$p_{ij} = \sum_{\alpha=\mathrm{A},\mathrm{B}} \big[p_{ij}^\alpha + \rho^\alpha \big(v_i^\alpha - v_i\big)\big(v_j^\alpha - v_j\big) \big] \quad (i, j = 1, 2), \tag{7a}$$

$$p_{33} = p_{33}^{\mathrm{A}} + p_{33}^{\mathrm{B}}, \tag{7b}$$

$$q_i = \sum_{\alpha=\mathrm{A},\mathrm{B}} \left[q_i^\alpha + p_{i1}^\alpha \big(v_1^\alpha - v_1\big) + p_{i2}^\alpha \big(v_2^\alpha - v_2\big) + \frac{3}{2} p^\alpha \big(v_i^\alpha - v_i\big) + \frac{1}{2}\rho^\alpha \big(v_i^\alpha - v_i\big) \big|\boldsymbol{v}^\alpha - \boldsymbol{v}\big|^2 \right] \quad (i = 1, 2). \tag{7c}$$

Finally, we introduce two additional macroscopic quantities: the concentration χ^α of species α and the flow velocity $\boldsymbol{w} = (w_1, w_2, 0)$ of the mixture based on the particle flow. They are defined as

$$\chi^\alpha = n^\alpha / n, \tag{8}$$

$$w_i = \chi^{\mathrm{A}} v_i^{\mathrm{A}} + \chi^{\mathrm{B}} v_i^{\mathrm{B}} \quad (i = 1, 2). \tag{9}$$

Note that the relation $\chi^{\mathrm{A}} + \chi^{\mathrm{B}} = 1$ holds by definition.

3.2. Dimensionless description

In this section, we reformulate the basic equation and its boundary condition in dimensionless form and clarify the parameters that characterize the problem.

Let us introduce the following dimensionless variables:

$$x_i = \big(2/\sqrt{\pi}\big)\big(X_i / l^{\mathrm{AA}}\big), \quad \zeta_i = \xi_i / c_{\mathrm{w}}, \quad f^\alpha = \big(c_{\mathrm{w}}^3 / n_{\mathrm{w}}^{\mathrm{A}}\big) F^\alpha,$$
$$\widehat{m}^\alpha = m^\alpha / m^{\mathrm{A}}, \qquad K^{\beta\alpha} = B_0^{\beta\alpha} / B_0^{\mathrm{AA}},$$

where

$$c_{\mathrm{w}} = \big(2kT_{\mathrm{w}} / m^{\mathrm{A}}\big)^{1/2}, \qquad n_{\mathrm{w}}^{\mathrm{A}} = p_{\mathrm{w}}^{\mathrm{A}} / kT_{\mathrm{w}},$$
$$B_0^{\beta\alpha} = \int M_{\mathrm{w}}^\beta(\boldsymbol{\xi}_*) M_{\mathrm{w}}^\alpha(\boldsymbol{\xi}) B^{\beta\alpha}\big(|\boldsymbol{e}\cdot\boldsymbol{V}|/V, V\big) \, \mathrm{d}\Omega(\boldsymbol{e}) \, \mathrm{d}\boldsymbol{\xi}_* \, \mathrm{d}\boldsymbol{\xi},$$
$$l^{\mathrm{AA}} = \frac{(8kT_{\mathrm{w}}/\pi m^{\mathrm{A}})^{1/2}}{n_{\mathrm{w}}^{\mathrm{A}} B_0^{\mathrm{AA}}},$$

and M_{w}^{α} is the Maxwellian defined by

$$M_{\mathrm{w}}^{\alpha}(\boldsymbol{\xi}) = \frac{1}{(2\pi kT_{\mathrm{w}}/m^{\alpha})^{3/2}} \exp\left(-\frac{m^{\alpha}|\boldsymbol{\xi}|^2}{2kT_{\mathrm{w}}}\right).$$

The quantity l^{AA} is the reference mean free path of the molecules based on the interaction between molecules of species A.[1] Then the Boltzmann equation (1) is rewritten as

$$\zeta_1 \frac{\partial f^{\alpha}}{\partial x_1} = \sum_{\beta=\mathrm{A,B}} K^{\beta\alpha} \hat{J}^{\beta\alpha}\big(f^{\beta}, f^{\alpha}\big), \tag{10}$$

with

$$\hat{J}^{\beta\alpha}(f,g) = \int \big(f'_* g' - f_* g\big) b^{\beta\alpha}\big(\big|\boldsymbol{e}\cdot\widehat{\boldsymbol{V}}\big|/\widehat{V}, \widehat{V}\big)\, \mathrm{d}\Omega(\boldsymbol{e})\, \mathrm{d}\boldsymbol{\zeta}_*, \tag{11}$$

where

$$f'_* = f\big(x_1, \boldsymbol{\zeta}'_*\big), \quad g' = g\big(x_1, \boldsymbol{\zeta}'\big), \quad f_* = f(x_1, \boldsymbol{\zeta}_*), \quad g = g(x_1, \boldsymbol{\zeta}),$$

$$\boldsymbol{\zeta}' = \boldsymbol{\zeta} + \frac{\hat{\mu}^{\beta\alpha}}{\widehat{m}^{\alpha}}\big(\boldsymbol{e}\cdot\widehat{\boldsymbol{V}}\big)\boldsymbol{e}, \qquad \boldsymbol{\zeta}'_* = \boldsymbol{\zeta}_* - \frac{\hat{\mu}^{\beta\alpha}}{\widehat{m}^{\beta}}\big(\boldsymbol{e}\cdot\widehat{\boldsymbol{V}}\big)\boldsymbol{e},$$

$$\widehat{\boldsymbol{V}} = \boldsymbol{\zeta}_* - \boldsymbol{\zeta}, \quad \widehat{V} = \big|\widehat{\boldsymbol{V}}\big|, \quad \hat{\mu}^{\beta\alpha} = \frac{2\widehat{m}^{\alpha}\widehat{m}^{\beta}}{\widehat{m}^{\alpha} + \widehat{m}^{\beta}}, \quad b^{\beta\alpha} = B^{\beta\alpha}/B_0^{\beta\alpha}.$$

The boundary conditions (3a) and (3b) for F^{α} are transformed into those for f^{α} as

$$f^{\alpha} = \frac{\hat{p}_{\mathrm{w}}^{\alpha}}{(\pi/\widehat{m}^{\alpha})^{3/2}} \exp\big(-\widehat{m}^{\alpha}|\boldsymbol{\zeta}|^2\big), \quad \zeta_1 > 0,\ x_1 = 0, \tag{12a}$$

$$f^{\alpha} = \frac{\hat{p}_{\infty}^{\alpha}}{(\pi/\widehat{m}^{\alpha})^{3/2}\widehat{T}_{\infty}^{5/2}} \exp\left(-\frac{\widehat{m}^{\alpha}|\boldsymbol{\zeta} - \hat{\boldsymbol{v}}_{\infty}|^2}{\widehat{T}_{\infty}}\right), \quad \text{as } x_1 \to \infty, \tag{12b}$$

where

$$\hat{p}_{\mathrm{w}}^{\alpha} = \frac{p_{\mathrm{w}}^{\alpha}}{p_{\mathrm{w}}^{\mathrm{A}}}, \quad \hat{p}_{\infty}^{\alpha} = \frac{p_{\infty}^{\alpha}}{p_{\mathrm{w}}^{\mathrm{A}}}, \quad \widehat{T}_{\infty} = \frac{T_{\infty}}{T_{\mathrm{w}}}, \quad \hat{\boldsymbol{v}}_{\infty} = \big(\hat{v}_{1\infty}, \hat{v}_{2\infty}, 0\big) = \frac{\boldsymbol{v}_{\infty}}{c_{\mathrm{w}}}. \tag{13}$$

From (10)–(13), the problem is seen to be characterized by the following parameters:

$$\frac{p_{\mathrm{w}}^{\mathrm{B}}}{p_{\mathrm{w}}^{\mathrm{A}}},\ \frac{p_{\infty}^{\mathrm{A}}}{p_{\mathrm{w}}^{\mathrm{A}}},\ \frac{p_{\infty}^{\mathrm{B}}}{p_{\mathrm{w}}^{\mathrm{A}}},\ \frac{T_{\infty}}{T_{\mathrm{w}}},\ \frac{v_{1\infty}}{\sqrt{2kT_{\mathrm{w}}/m^{\mathrm{A}}}},\ \frac{v_{2\infty}}{\sqrt{2kT_{\mathrm{w}}/m^{\mathrm{A}}}},\ \frac{m^{\mathrm{B}}}{m^{\mathrm{A}}},\ K^{\mathrm{BB}},\ K^{\mathrm{BA}}\ \big(= K^{\mathrm{AB}}\big).$$

The relation $K^{\mathrm{BA}} = K^{\mathrm{AB}}$ comes from the symmetry property of $B^{\beta\alpha}$ with respect to the superscripts.

Dimensionless macroscopic quantities will also be used in the following sections. They are denoted by the notation of the corresponding dimensional ones with ˆ. That is

$$\big(\hat{n}^{\alpha}, \hat{n}\big) = \big(n^{\alpha}, n\big)/n_{\mathrm{w}}^{\mathrm{A}}, \quad \hat{\rho} = \rho/m^{\mathrm{A}}n_{\mathrm{w}}^{\mathrm{A}}, \quad \big(\hat{\boldsymbol{v}}^{\alpha}, \hat{\boldsymbol{v}}, \widehat{\boldsymbol{w}}\big) = \big(\boldsymbol{v}^{\alpha}, \boldsymbol{v}, \boldsymbol{w}\big)/c_{\mathrm{w}},$$

$$\big(\hat{p}^{\alpha}, \hat{p}\big) = \big(p^{\alpha}, p\big)/p_{\mathrm{w}}^{\mathrm{A}}, \quad \big(\widehat{T}^{\alpha}, \widehat{T}\big) = \big(T^{\alpha}, T\big)/T_{\mathrm{w}},$$

$$\big(\hat{p}_{ij}^{\alpha}, \hat{p}_{ij}\big) = \big(p_{ij}^{\alpha}, p_{ij}\big)/p_{\mathrm{w}}^{\mathrm{A}}, \quad \big(\hat{q}_i^{\alpha}, \hat{q}_i\big) = \big(q_i^{\alpha}, q_i\big)/p_{\mathrm{w}}^{\mathrm{A}}c_{\mathrm{w}}.$$

The expressions of these quantities in terms of the moment of f^{α} can be easily obtained from (3)–(9) and thus are omitted here.

Before closing this section, we remark that the quantity $B_0^{\beta\alpha}$ is not necessarily finite if the intermolecular potential extends to infinity. In the case, $B_0^{\beta\alpha}$ should be replaced by an appropriate quantity. The choice of such a quantity is a matter of taste. For instance, using $B_0^{\beta\alpha}$ calculated by the same intermolecular potential with a radial or angular cutoff is a possible candidate. The reader is referred to Appendix A.2 in [13] for another possible choice, which would be practically more preferable.

[1] For hard sphere gases, $l^{\mathrm{AA}} = (\sqrt{2}\pi(d^{\mathrm{A}})^2 n_{\mathrm{w}}^{\mathrm{A}})^{-1}$, where d^{A} is the diameter of a molecule of species A.

4. Asymptotic analysis

In this section, we carry out a systematic asymptotic analysis of the boundary-value problem for f^α [Eqs. (10) and (12)] in the situation where evaporation or condensation takes place only weakly:

$$\left|\hat{v}_{1\infty}\right| = \epsilon \ll 1. \tag{14}$$

There is no geometrical characteristic length in the problem, and apparently the solution has the length scale of variation of the order of the mean free path l^{AA}. Nevertheless, we will consider a solution having another length scale of variation, i.e., a slowly varying solution with the length scale of l^{AA}/ϵ. The idea of the slowly varying solution was first introduced by Sone [7] in the study of the corresponding problem for a single-component vapor. Physically, the idea comes from the fact that a boundary layer, what is called the suction boundary layer, with the thickness of the order of the kinematic viscosity divided by the suction speed ($\sim l^{\mathrm{AA}}/\epsilon$) appears near a body surface with suction. The reader is referred to [8] for the description of the layer in the classical macroscopic framework and to [1] for that in the framework of kinetic theory.

4.1. Slowly varying solution

We consider a slowly varying solution [$\partial f^\alpha/\partial x_1 = \mathrm{O}(f^\alpha \epsilon)$] of the Boltzmann equation (10). We rescale the equation, by introducing a new space coordinate $y = x_1\epsilon$, as

$$\zeta_1 \frac{\partial f^\alpha}{\partial y} = \frac{1}{\epsilon} \sum_{\beta=\mathrm{A,B}} K^{\beta\alpha} \hat{J}^{\beta\alpha}\big(f^\beta, f^\alpha\big), \tag{15}$$

and seek the solution in a power series of ϵ:

$$f^\alpha = f^\alpha_{(0)} + f^\alpha_{(1)}\epsilon + \cdots. \tag{16}$$

Correspondingly, we expand the macroscopic quantities as

$$h^\alpha = h^\alpha_{(0)} + h^\alpha_{(1)}\epsilon + \cdots, \qquad h = h_{(0)} + h_{(1)}\epsilon + \cdots,$$

where $h = \hat{n}, \hat{p}, \widehat{T}, \hat{v}_1, \hat{v}_2$, etc. and assume that $\hat{v}_{1(0)} = \hat{v}^\alpha_{1(0)} = \hat{w}_{1(0)} = 0$, taking account of the physical situation (14) under consideration. The component functions $h^\alpha_{(m)}$ and $h_{(m)}$ are expressed in terms of the moments of $f^\alpha_{(n)}$ with $n \leqslant m$. A part of their explicit forms are given in Appendix A. Substituting (16) into (15) gives a series of integral equations for the component functions $f^\alpha_{(n)}$ ($n = 0, 1, 2, \ldots$):

$$\sum_{\beta=\mathrm{A,B}} K^{\beta\alpha} \hat{J}^{\beta\alpha}\big(f^\beta_{(0)}, f^\alpha_{(0)}\big) = 0, \tag{17a}$$

$$\sum_{\beta=\mathrm{A,B}} K^{\beta\alpha}\big[\hat{J}^{\beta\alpha}\big(f^\beta_{(1)}, f^\alpha_{(0)}\big) + \hat{J}^{\beta\alpha}\big(f^\beta_{(0)}, f^\alpha_{(1)}\big)\big] = \zeta_1 \frac{\partial f^\alpha_{(0)}}{\partial y}, \tag{17b}$$

$$\sum_{\beta=\mathrm{A,B}} K^{\beta\alpha}\big[\hat{J}^{\beta\alpha}\big(f^\beta_{(m)}, f^\alpha_{(0)}\big) + \hat{J}^{\beta\alpha}\big(f^\beta_{(0)}, f^\alpha_{(m)}\big)\big]$$
$$= \zeta_1 \frac{\partial f^\alpha_{(m-1)}}{\partial y} - \sum_{n=1}^{m-1} \sum_{\beta=\mathrm{A,B}} K^{\beta\alpha} \hat{J}^{\beta\alpha}\big(f^\beta_{(n)}, f^\alpha_{(m-n)}\big) \quad (m = 2, 3, \ldots). \tag{17c}$$

The solution of (17a) is a local equilibrium distribution at rest, which reads

$$f^\alpha_{(0)} = \frac{\hat{n}^\alpha_{(0)}}{\widehat{T}^{3/2}_{(0)}} \left(\frac{\widehat{m}^\alpha}{\pi}\right)^{3/2} \exp\left(-\frac{\widehat{m}^\alpha[\zeta_1^2 + (\zeta_2 - \hat{v}_{2(0)})^2 + \zeta_3^2]}{\widehat{T}_{(0)}}\right). \tag{18}$$

Note that, for this distribution, the temperature is common to species, i.e.,

$$\widehat{T}^\alpha_{(0)} = \widehat{T}_{(0)},$$

and the following relations hold:

$$\hat{p}^{\alpha}_{ij(0)} = \hat{p}^{\alpha}_{(0)}\delta_{ij}, \quad \hat{p}_{ij(0)} = \hat{p}_{(0)}\delta_{ij}, \quad \hat{q}^{\alpha}_{i(0)} = \hat{q}_{i(0)} = 0,$$

where δ_{ij} is Kronecker's delta.

Eq. (17b) is an inhomogeneous linear integral equation for $f^{\alpha}_{(1)}$. Hence in order that the solution exists, the solvability condition must be satisfied, which reads

$$\int \zeta_1 \frac{\partial f^{\alpha}_{(0)}}{\partial y}\, \mathrm{d}\boldsymbol{\zeta} = 0, \tag{19a}$$

$$\int \sum_{\alpha=\mathrm{A,B}} \zeta_1 \begin{bmatrix} \widehat{m}^{\alpha}\zeta_i \\ \widehat{m}^{\alpha}|\boldsymbol{\zeta}|^2 \end{bmatrix} \frac{\partial f^{\alpha}_{(0)}}{\partial y}\, \mathrm{d}\boldsymbol{\zeta} = 0. \tag{19b}$$

Eq. (19) is no other than the conservation laws of the mass, momentum, and energy at $\mathrm{O}(\epsilon^0)$. Substitution of (18) results in five degenerate equations and one equation for $\hat{p}_{(0)}$:

$$\frac{\mathrm{d}\hat{p}_{(0)}}{\mathrm{d}y} = 0. \tag{20}$$

Under this condition, Eq. (17b) is solved to yield

$$\begin{aligned} f^{\alpha}_{(1)} = f^{\alpha}_{(0)}\Bigg[&\frac{\hat{p}^{\alpha}_{(1)}}{\hat{p}^{\alpha}_{(0)}} + \frac{2\widehat{m}^{\alpha}}{\sqrt{\widehat{T}_{(0)}}}\big(\hat{v}_{1(1)}C_1 + \hat{v}_{2(1)}C_2\big) + \frac{\widehat{T}_{(1)}}{\widehat{T}_{(0)}}\left(\widehat{m}^{\alpha}|\boldsymbol{C}|^2 - \frac{5}{2}\right) \\ &- \frac{1}{\hat{n}_{(0)}}\left(C_1\mathcal{D}^{\alpha}\big(|\boldsymbol{C}|\big)\frac{\mathrm{d}\chi^{\mathrm{A}}_{(0)}}{\mathrm{d}y} + C_1\mathcal{A}^{\alpha}\big(|\boldsymbol{C}|\big)\frac{1}{\widehat{T}_{(0)}}\frac{\mathrm{d}\widehat{T}_{(0)}}{\mathrm{d}y} + C_1C_2\mathcal{B}^{\alpha}\big(|\boldsymbol{C}|\big)\frac{1}{\widehat{T}^{1/2}_{(0)}}\frac{\mathrm{d}\hat{v}_{2(0)}}{\mathrm{d}y}\right)\Bigg], \end{aligned} \tag{21}$$

where $\boldsymbol{C} = (\boldsymbol{\zeta} - \hat{\boldsymbol{v}}_{(0)})/\sqrt{\widehat{T}_{(0)}}$ with $\hat{\boldsymbol{v}}_{(0)} = (0, \hat{v}_{2(0)}, 0)$. The derivation is given in Appendix B with the definition of the functions $\mathcal{A}^{\alpha}$, $\mathcal{B}^{\alpha}$, and $\mathcal{D}^{\alpha}$ [see (B.4)–(B.6)].

Eq. (17c) can be solved in the same way. It is an inhomogeneous linear integral equation for f^{α}_{Hm} ($m = 2, 3, \ldots$) and is to be solved successively in increasing order of m, provided that the following solvability condition is satisfied:

$$\int \zeta_1 \frac{\partial f^{\alpha}_{(m-1)}}{\partial y}\, \mathrm{d}\boldsymbol{\zeta} = 0, \tag{22a}$$

$$\int \sum_{\alpha=\mathrm{A,B}} \zeta_1 \begin{bmatrix} \widehat{m}^{\alpha}\zeta_i \\ \widehat{m}^{\alpha}|\boldsymbol{\zeta}|^2 \end{bmatrix} \frac{\partial f^{\alpha}_{(m-1)}}{\partial y}\, \mathrm{d}\boldsymbol{\zeta} = 0. \tag{22b}$$

Here the first line of (22b) with $i = 3$ always degenerates because f^{α} is even in ζ_3. Eq. (22) is the conservation laws of the mass, momentum, and energy at $\mathrm{O}(\epsilon^{m-1})$. For $m = 2$, it yields

$$\frac{\mathrm{d}\hat{n}^{\alpha}_{(0)}\hat{v}^{\alpha}_{1(1)}}{\mathrm{d}y} = 0, \tag{23a}$$

$$\frac{\mathrm{d}\hat{p}_{(1)}}{\mathrm{d}y} = 0, \tag{23b}$$

$$\frac{\mathrm{d}}{\mathrm{d}y}\left(\frac{1}{2}\hat{p}_{12(1)} + \hat{\rho}_{(0)}\hat{v}_{1(1)}\hat{v}_{2(0)}\right) = 0, \tag{23c}$$

$$\frac{\mathrm{d}}{\mathrm{d}y}\left(\hat{q}_{1(1)} + \frac{5}{2}\hat{p}_{(0)}\hat{v}_{1(1)} + \hat{p}_{12(1)}\hat{v}_{2(0)} + \hat{\rho}_{(0)}\hat{v}_{1(1)}\hat{v}^2_{2(0)}\right) = 0, \tag{23d}$$

with

$$\hat{v}^{\mathrm{A}}_{1(1)} - \hat{v}^{\mathrm{B}}_{1(1)} = -\frac{\widehat{D}_{\mathrm{AB}}}{\chi^{\mathrm{A}}_{(0)}\chi^{\mathrm{B}}_{(0)}} \frac{\widehat{T}^{1/2}_{(0)}}{\hat{n}_{(0)}} \left(\frac{\mathrm{d}\chi^{A}_{(0)}}{\mathrm{d}y} + \frac{k_{\mathrm{T}}}{\widehat{T}_{(0)}} \frac{\mathrm{d}\widehat{T}_{(0)}}{\mathrm{d}y} \right), \tag{24}$$

$$\hat{p}_{12(1)} = -\hat{\mu}\widehat{T}^{1/2}_{(0)} \frac{\mathrm{d}\hat{v}_{2(0)}}{\mathrm{d}y}, \tag{25}$$

$$\hat{q}_{1(1)} = -\hat{\lambda}\widehat{T}^{1/2}_{(0)} \frac{\mathrm{d}\widehat{T}_{(0)}}{\mathrm{d}y} + k_{\mathrm{T}}\, \hat{p}_{(0)} \big(\hat{v}^{\mathrm{A}}_{1(1)} - \hat{v}^{\mathrm{B}}_{1(1)}\big) + \frac{5}{2} \hat{p}_{(0)} \big(\widehat{w}_{1(1)} - \hat{v}_{1(1)}\big), \tag{26}$$

where

$$\widehat{D}_{\mathrm{AB}} = \frac{4\pi}{3} \chi^{\mathrm{A}}_{(0)}\chi^{\mathrm{B}}_{(0)} \int_0^\infty C^4 \big(\mathcal{D}^{\mathrm{A}} E^{\mathrm{A}} - \mathcal{D}^{\mathrm{B}} E^{\mathrm{B}}\big)\, \mathrm{d}C \,(>0), \tag{27a}$$

$$\widehat{D}_{\mathrm{T}} = \frac{4\pi}{3} \chi^{\mathrm{A}}_{(0)}\chi^{\mathrm{B}}_{(0)} \int_0^\infty C^4 \big(\mathcal{A}^{\mathrm{A}} E^{\mathrm{A}} - \mathcal{A}^{\mathrm{B}} E^{\mathrm{B}}\big)\, \mathrm{d}C = \frac{4\pi}{3} \chi^{\mathrm{A}}_{(0)}\chi^{\mathrm{B}}_{(0)} \sum_{\alpha=\mathrm{A},\mathrm{B}} \chi^{\alpha}_{(0)} \int_0^\infty C^4 \left(\widehat{m}^{\alpha} C^2 - \frac{5}{2} \right) \mathcal{D}^{\alpha} E^{\alpha}\, \mathrm{d}C, \tag{27b}$$

$$k_{\mathrm{T}} = \frac{\widehat{D}_{\mathrm{T}}}{\widehat{D}_{\mathrm{AB}}} = \frac{\int_0^\infty C^4 (\mathcal{A}^{\mathrm{A}} E^{\mathrm{A}} - \mathcal{A}^{\mathrm{B}} E^{\mathrm{B}})\, \mathrm{d}C}{\int_0^\infty C^4 (\mathcal{D}^{\mathrm{A}} E^{\mathrm{A}} - \mathcal{D}^{\mathrm{B}} E^{\mathrm{B}})\, \mathrm{d}C}, \tag{27c}$$

$$\hat{\mu} = \frac{8\pi}{15} \sum_{\alpha=\mathrm{A},\mathrm{B}} \widehat{m}^{\alpha} \chi^{\alpha}_{(0)} \int_0^\infty C^6 \mathcal{B}^{\alpha} E^{\alpha}\, \mathrm{d}C \,(>0), \tag{27d}$$

$$\hat{\lambda} = \frac{4\pi}{3} \sum_{\alpha=\mathrm{A},\mathrm{B}} \chi^{\alpha}_{(0)} \int_0^\infty C^4 \left(\widehat{m}^{\alpha} C^2 - \frac{5}{2} \right) \mathcal{A}^{\alpha} E^{\alpha}\, \mathrm{d}C - \frac{k_{\mathrm{T}}^2 \widehat{D}_{\mathrm{AB}}}{\chi^{\mathrm{A}}_{(0)}\chi^{\mathrm{B}}_{(0)}} \;(>0). \tag{27e}$$

The quantities $\widehat{D}_{\mathrm{AB}}$, $\widehat{D}_{\mathrm{T}}$, k_{T}, $\hat{\mu}$, and $\hat{\lambda}$ are functions of $\chi^{\mathrm{A}}_{(0)}$ and $\widehat{T}_{(0)}$ as well as the parameters $m^{\mathrm{B}}/m^{\mathrm{A}}$, K^{AB}, and K^{BB}. The positivity of $\widehat{D}_{\mathrm{AB}}$, $\hat{\mu}$, and $\hat{\lambda}$ is due to the symmetry property of the collision operator. The different expressions for $\widehat{D}_{\mathrm{T}}$ are also due to this property. (See Appendix B.2.) Incidentally, these coefficients are related to the mutual-diffusion coefficient D_{AB}, the viscosity μ, the thermal conductivity λ, and the thermal-diffusion coefficient D_{T} in the following way,

$$D_{\mathrm{AB}}\big(n, T, \chi^{\mathrm{A}}\big) = \frac{\sqrt{2kT_{\mathrm{w}}/m^{\mathrm{A}}}}{B_0^{\mathrm{AA}}} \frac{\sqrt{2kT/m^{\mathrm{A}}}}{n} \widehat{D}_{\mathrm{AB}}\big(\widehat{T}, \chi^{\mathrm{A}}\big), \tag{28a}$$

$$\mu\big(T, \chi^{\mathrm{A}}\big) = \frac{m^{\mathrm{A}}}{2} \frac{\sqrt{2kT_{\mathrm{w}}/m^{\mathrm{A}}}}{B_0^{\mathrm{AA}}} \sqrt{2kT/m^{\mathrm{A}}}\, \hat{\mu}\big(\widehat{T}, \chi^{\mathrm{A}}\big), \tag{28b}$$

$$\lambda\big(T, \chi^{\mathrm{A}}\big) = \frac{k\sqrt{2kT_{\mathrm{w}}/m^{\mathrm{A}}}}{B_0^{\mathrm{AA}}} \sqrt{2kT/m^{\mathrm{A}}}\, \hat{\lambda}\big(\widehat{T}, \chi^{\mathrm{A}}\big), \tag{28c}$$

$$D_{\mathrm{T}}\big(n, T, \chi^{\mathrm{A}}\big) = \frac{\sqrt{2kT_{\mathrm{w}}/m^{\mathrm{A}}}}{B_0^{\mathrm{AA}}} \frac{\sqrt{2kT/m^{\mathrm{A}}}}{n} \widehat{D}_{\mathrm{T}}\big(\widehat{T}, \chi^{\mathrm{A}}\big), \tag{28d}$$

and k_{T} is the thermo-diffusion ratio in Chapman and Cowling [14]. Approximate expressions for these coefficients are available in the literature [14–16]. In the case of hard sphere gases, $\widehat{D}_{\mathrm{AB}}$, $\hat{\mu}$, $\hat{\lambda}$, and $\widehat{D}_{\mathrm{T}}$ (or k_{T}) are all independent of $\widehat{T}$, and their highly accurate data are also available [17].

Eqs. (20) and (23) with (24)–(26) form a closed system for $\hat{p}_{(0)}$, $\chi^{\mathrm{A}}_{(0)}$, $\widehat{T}_{(0)}$, $\hat{v}_{1(1)}$, $\hat{v}_{2(0)}$, and $\hat{p}_{(1)}$. Continuation of the above process results in the systems for macroscopic quantities at higher orders. We will not present these here because they are required only in the discussion of the evaporation case under a certain simplified situation (Section 5.2.2).

4.2. Boundary condition at the interface

Eqs. (20) and (23) with (24)–(26) derived in Section 4.1 form a fluid-dynamic set of equations for the component functions $\hat{p}_{(0)}$, $\chi^{\mathrm{A}}_{(0)}$, $\widehat{T}_{(0)}$, $\hat{v}_{1(1)}$, $\hat{v}_{2(0)}$, and $\hat{p}_{(1)}$ that describes the overall behavior of the mixture. In its derivation, however, the kinetic boundary condition (12a) was not taken into account. We will study the compatibility of the slowly varying solution with the kinetic boundary condition and derive the boundary condition at the interface for the fluid-dynamic set.

Fortunately, at the leading order, the slowly varying solution $f^{\alpha}_{(0)}$ can be matched with (12a) by setting

$$\hat{p}^{\mathrm{A}}_{(0)} = 1, \quad \hat{p}^{\mathrm{B}}_{(0)} = \hat{p}^{\mathrm{B}}_{\mathrm{w}}, \quad \hat{v}_{2(0)} = 0, \quad \widehat{T}_{(0)} = 1, \quad \text{at } y = 0. \tag{29}$$

This is the boundary condition for the macroscopic quantities of $\mathrm{O}(\epsilon^0)$.

Next, we proceed to the first order to obtain the boundary condition for the macroscopic quantities of $\mathrm{O}(\epsilon^1)$. Obviously, the slowly varying solution cannot be matched with the condition (12a) at this order, because there is a flow of $\mathrm{O}(\epsilon)$ in the situation under consideration [see (14)]. Hence, we introduce a correction in the vicinity of the interface whose scale of variation is the mean free path of a molecule. Let us express the solution in this layer in the form $f^{\alpha}_{\mathrm{S}} + f^{\alpha}_{\mathrm{K}}$, where f^{α}_{S} denotes the slowly varying solution (f^{α} in Section 4.1) and f^{α}_{K} the correction to it. The latter is supposed to be appreciable only in the layer and vanish rapidly as a function of the original dimensionless coordinate x_1. Since the slowly varying solution matches the kinetic boundary condition at the leading order, the correction is required from the first order. Hence, f^{α}_{K} is expanded in a power series of ϵ as

$$f^{\alpha}_{\mathrm{K}} = f^{\alpha}_{\mathrm{K}(1)}\epsilon + f^{\alpha}_{\mathrm{K}(2)}\epsilon^2 + \cdots. \tag{30}$$

Correspondingly, we denote a macroscopic quantity h in the correction layer by $h_{\mathrm{S}} + h_{\mathrm{K}}$, where h_{S} represents the slowly varying solution, and expand h_{K} in the same way: $h_{\mathrm{K}} = h_{\mathrm{K}(1)}\epsilon + h_{\mathrm{K}(2)}\epsilon^2 + \cdots$. Substituting $f^{\alpha} = f^{\alpha}_{\mathrm{S}} + f^{\alpha}_{\mathrm{K}}$ into (10) and taking into account the expansions (16) with $f^{\alpha} = f^{\alpha}_{\mathrm{S}}$ and (30) yield a series of boundary-value problems for $f^{\alpha}_{\mathrm{K}(m)}$ ($m = 1, 2, \ldots$). For the present purpose, the analysis of the problem for $f^{\alpha}_{\mathrm{K}(1)}$ is sufficient. This problem reads

$$\zeta_1 \frac{\partial f^{\alpha}_{\mathrm{K}(1)}}{\partial x_1} = \sum_{\beta=\mathrm{A},\mathrm{B}} K^{\beta\alpha}\big[\hat{J}^{\beta\alpha}\big(f^{\beta}_{\mathrm{S}(0)}, f^{\alpha}_{\mathrm{K}(1)}\big) + \hat{J}^{\beta\alpha}\big(f^{\beta}_{\mathrm{K}(1)}, f^{\alpha}_{\mathrm{S}(0)}\big)\big], \tag{31}$$

$$f^{\alpha}_{\mathrm{K}(1)} = -f^{\alpha}_{\mathrm{S}(1)}, \quad \text{for } \zeta_1 > 0, \text{ at } x_1 = 0, \tag{32}$$

$$f^{\alpha}_{\mathrm{K}(1)} \to 0, \quad \text{as } x_1 \to \infty, \tag{33}$$

where the quantities with subscript S in (31) and (32) represent their values at $x_1 = 0$. Taking into account (29), this problem is transformed into

$$\zeta_1 \frac{\partial \Phi^{\alpha}}{\partial z} = \sum_{\beta=\mathrm{A},\mathrm{B}} K^{\beta\alpha}\chi^{\beta}_{\mathrm{w}}\mathcal{L}^{\beta\alpha}\big(\Phi^{\beta}, \Phi^{\alpha}\big), \tag{34a}$$

$$\begin{aligned}\Phi^{\alpha} = &-\bigg[\frac{\hat{p}^{\alpha}_{(1)}}{\hat{p}^{\alpha}_{\mathrm{w}}} + 2\widehat{m}^{\alpha}\big(\hat{v}_{1(1)}\zeta_1 + \hat{v}_{2(1)}\zeta_2\big) + \widehat{T}_{(1)}\bigg(\widehat{m}^{\alpha}|\boldsymbol{\zeta}|^2 - \frac{5}{2}\bigg)\\ &- \frac{1}{\hat{n}_{\mathrm{w}}}\bigg(\zeta_1\mathcal{D}^{\alpha}\big(|\boldsymbol{\zeta}|\big)\frac{\mathrm{d}\chi^{\mathrm{A}}_{(0)}}{\mathrm{d}y} + \zeta_1\mathcal{A}^{\alpha}\big(|\boldsymbol{\zeta}|\big)\frac{\mathrm{d}\widehat{T}_{(0)}}{\mathrm{d}y} + \zeta_1\zeta_2\mathcal{B}^{\alpha}\big(|\boldsymbol{\zeta}|\big)\frac{\mathrm{d}\hat{v}_{2(0)}}{\mathrm{d}y}\bigg)\bigg],\\ &\text{for } \zeta_1 > 0, \quad \text{at } z = 0,\end{aligned} \tag{34b}$$

$$\Phi^{\alpha} \to 0, \quad \text{as } z \to \infty, \tag{34c}$$

where $\Phi^{\alpha} = f^{\alpha}_{\mathrm{K}(1)}/[\hat{p}^{\alpha}_{\mathrm{w}}E^{\alpha}(|\boldsymbol{\zeta}|)]$, $z = \hat{n}_{\mathrm{w}}x_1$, $\chi^{\alpha}_{\mathrm{w}} = p^{\alpha}_{\mathrm{w}}/p_{\mathrm{w}}[= \hat{p}^{\alpha}_{\mathrm{w}}/(1 + \hat{p}^{\mathrm{B}}_{\mathrm{w}})]$, and $\mathcal{L}^{\beta\alpha}$ is $\mathcal{L}^{\beta\alpha}_a$ with $a = 1$ [see (B.3) for the definition of $\mathcal{L}^{\beta\alpha}_a$]. The functions $\mathcal{A}^{\alpha}$, $\mathcal{B}^{\alpha}$, and $\mathcal{D}^{\alpha}$ are those for $\widehat{T}_{(0)} = 1$ and $\chi^{\alpha}_{(0)} = \chi^{\alpha}_{\mathrm{w}}$ [see (B.4)–(B.6)]. In the meantime, it is proved in [18] that the half-space problem

$$\zeta_1 \frac{\partial \Phi^{\alpha}}{\partial z} = \sum_{\beta=\mathrm{A},\mathrm{B}} K^{\beta\alpha}\chi^{\beta}_{\mathrm{w}}\mathcal{L}^{\beta\alpha}\big(\Phi^{\beta}, \Phi^{\alpha}\big), \tag{35a}$$

$$\Phi^\alpha = a_0^\alpha + 2\widehat{m}^\alpha a_2\zeta_2 + \widehat{m}^\alpha a_4|\boldsymbol{\zeta}|^2 + g^\alpha(\boldsymbol{\zeta}), \quad \zeta_1 > 0, \quad \text{at } z = 0, \tag{35b}$$

$$\Phi^\alpha \to 0, \quad \text{as } z \to \infty, \tag{35c}$$

with g^α being a given function and a_0^α, a_2, and a_4 being undetermined constants, has a solution if and only if the constants a_0^α, a_2, and a_4 take special values and the solution is unique.[2] Because of the symmetry property of $\mathcal{L}^{\beta\alpha}$, we can seek the solution as an even function of ζ_2 when g^α is even in ζ_2 and as an odd function of ζ_2 when g^α is odd in ζ_2. Therefore we may suppose that $a_2 = 0$ for g^α even in ζ_2 and $a_0^\alpha = a_4 = 0$ for g^α odd in ζ_2. Keeping these in mind, let us denote a_2 by $a_\parallel$ for $g^\alpha = \zeta_1\zeta_2\mathcal{B}^\alpha$, and denote $a_0^\alpha + \frac{5}{2}a_4$ and a_4 by a_V^α and c_V for $g^\alpha = -2\widehat{m}^\alpha\zeta_1$, by a_T^α and c_T for $g^\alpha = \zeta_1\mathcal{A}^\alpha$, and by a_χ^α and c_χ for $g^\alpha = \zeta_1\mathcal{D}^\alpha$, respectively. Then, because (34) is linear, we can express $\hat{p}_{(1)}^\alpha$, $\hat{v}_{2(1)}$, and $\widehat{T}_{(1)}$ at the interface ($z = 0$) as follows:

$$-\frac{\hat{p}_{(1)}^\alpha}{\hat{p}_\mathrm{w}^\alpha} = a_\mathrm{V}^\alpha\hat{v}_{1(1)} + a_\mathrm{T}^\alpha\frac{1}{\hat{n}_\mathrm{w}}\frac{\mathrm{d}\widehat{T}_{(0)}}{\mathrm{d}y} + a_\chi^\alpha\frac{1}{\hat{n}_\mathrm{w}}\frac{\mathrm{d}\chi_{(0)}^\mathrm{A}}{\mathrm{d}y}, \tag{36}$$

$$-\hat{v}_{2(1)} = a_\parallel\frac{1}{\hat{n}_\mathrm{w}}\frac{\mathrm{d}\hat{v}_{2(0)}}{\mathrm{d}y}, \tag{37}$$

$$-\widehat{T}_{(1)} = c_\mathrm{V}\hat{v}_{1(1)} + c_\mathrm{T}\frac{1}{\hat{n}_\mathrm{w}}\frac{\mathrm{d}\widehat{T}_{(0)}}{\mathrm{d}y} + c_\chi\frac{1}{\hat{n}_\mathrm{w}}\frac{\mathrm{d}\chi_{(0)}^\mathrm{A}}{\mathrm{d}y}. \tag{38}$$

For the later convenience, using the relation

$$\hat{v}_{1(1)} = \widehat{w}_{1(1)} + \frac{\widehat{m}^\mathrm{B} - 1}{\hat{n}_{(0)}^\mathrm{A} + \widehat{m}^\mathrm{B}\hat{n}_{(0)}^\mathrm{B}}\widehat{D}_\mathrm{AB}\widehat{T}_{(0)}^{1/2}\left(\frac{\mathrm{d}\chi_{(0)}^\mathrm{A}}{\mathrm{d}y} + \frac{k_\mathrm{T}}{\widehat{T}_{(0)}}\frac{\mathrm{d}\widehat{T}_{(0)}}{\mathrm{d}y}\right),$$

which is readily derived from (24) and the definitions of $\hat{v}_{1(1)}$ and $\widehat{w}_{1(1)}$ (see Appendix A.2), we rewrite (36) and (38) as

$$-\frac{\hat{p}_{(1)}^\alpha}{\hat{p}_\mathrm{w}^\alpha} = a_\mathrm{V}^\alpha\widehat{w}_{1(1)} + \left(a_\mathrm{V}^\alpha\widehat{D}_\mathrm{AB}\frac{\widehat{m}^\mathrm{B} - 1}{\widehat{m}_\mathrm{w}} + a_\chi^\alpha\right)\frac{1}{\hat{n}_\mathrm{w}}\frac{\mathrm{d}\chi_{(0)}^\mathrm{A}}{\mathrm{d}y} + \left(a_\mathrm{V}^\alpha\widehat{D}_\mathrm{T}\frac{\widehat{m}^\mathrm{B} - 1}{\widehat{m}_\mathrm{w}} + a_\mathrm{T}^\alpha\right)\frac{1}{\hat{n}_\mathrm{w}}\frac{d\widehat{T}_{(0)}}{\mathrm{d}y}, \tag{39}$$

$$-\widehat{T}_{(1)} = c_\mathrm{V}\widehat{w}_{1(1)} + \left(c_\mathrm{V}\widehat{D}_\mathrm{AB}\frac{\widehat{m}^\mathrm{B} - 1}{\widehat{m}_\mathrm{w}} + c_\chi\right)\frac{1}{\hat{n}_\mathrm{w}}\frac{\mathrm{d}\chi_{(0)}^\mathrm{A}}{\mathrm{d}y} + \left(c_\mathrm{V}\widehat{D}_\mathrm{T}\frac{\widehat{m}^\mathrm{B} - 1}{\widehat{m}_\mathrm{w}} + c_\mathrm{T}\right)\frac{1}{\hat{n}_\mathrm{w}}\frac{\mathrm{d}\widehat{T}_{(0)}}{\mathrm{d}y}, \tag{40}$$

where

$$\widehat{m}_\mathrm{w} = \chi_\mathrm{w}^\mathrm{A} + \widehat{m}^\mathrm{B}\chi_\mathrm{w}^\mathrm{B}.$$

Note that $a_\parallel$, a_V^α, a_T^α, a_χ^α, c_V, c_T, and c_χ are constants depending on χ_w^A and that $\widehat{D}_\mathrm{AB}$ and $\widehat{D}_\mathrm{T}$ are evaluated with $\chi_{(0)}^\mathrm{A} = \chi_\mathrm{w}^A$ and $\widehat{T}_{(0)} = 1$. Summation of (39) multiplied by $\hat{p}_\mathrm{w}^\alpha$ with respect to α ($\alpha = \mathrm{A}, \mathrm{B}$) gives the relation among $\hat{p}_{(1)}$, $\widehat{w}_{1(1)}$, and the gradients of $\chi_{(0)}^\mathrm{A}$ and $\widehat{T}_{(0)}$ at the interface, i.e.,

$$\hat{p}_{(1)} = -a_\mathrm{V}\hat{p}_\mathrm{w}\widehat{w}_{1(1)} - \left(a_\mathrm{V}\widehat{D}_\mathrm{T}\frac{\widehat{m}^\mathrm{B} - 1}{\widehat{m}_\mathrm{w}} + a_\mathrm{T}\right)\frac{\mathrm{d}\widehat{T}_{(0)}}{\mathrm{d}y} - \left(a_\mathrm{V}\widehat{D}_\mathrm{AB}\frac{\widehat{m}^\mathrm{B} - 1}{\widehat{m}_\mathrm{w}} + a_\chi\right)\frac{\mathrm{d}\chi_{(0)}^\mathrm{A}}{\mathrm{d}y}, \quad \text{at } y = 0, \tag{41}$$

where

$$a_I = a_I^\mathrm{A}\chi_\mathrm{w}^\mathrm{A} + a_I^\mathrm{B}\chi_\mathrm{w}^\mathrm{B} \quad (I = \mathrm{V}, \mathrm{T}, \chi).$$

Eqs. (29) and (41) are the boundary conditions for the fluid-dynamic equations (20) and (23) at the interface.

The coefficients $a_\parallel$, a_V^α, c_V, etc. occurring in (36)–(38) are the so-called slip and jump coefficients. Their approximate expressions are available in the literature (e.g. [19–21]). In the case of hard sphere gases, accurate data of those coefficients have been obtained recently as a consequence of accurate finite-difference analyses of the system (35) [22–24].[3]

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[2] Strictly, the theorem was proved only for hard-sphere gases in the case of mixture. The proof for the other molecular models is still an open problem. Incidentally, in the case of single-species gas, the proof is given for more general molecular models (see the references in [18]).

[3] The correspondence of notations would be in order. $a_\parallel$ is $-b$ in [22], a_V^α and c_V are, respectively, $-\gamma^\alpha$ and $-\delta$ in [23], and a_T^α, a_χ^α, c_T, and c_χ are, respectively, $-\gamma_\mathrm{T}^\alpha$, $-\gamma_\chi^\alpha$, $-\delta_\mathrm{T}$, and $-\delta_\chi$ in [24]. X_0^A in [22–24] is $\chi_\mathrm{w}^\mathrm{A}$ in this paper.

4.3. Summary

In Section 4, we have obtained the fluid-dynamic equations [(20) and (23) with (24)–(26)] and their boundary conditions [(29) and (41)]. This fluid-dynamic system describes the behavior of the leading-order macroscopic quantities $\chi^{\mathrm{A}}_{(0)}$, $\hat{v}_{2(0)}$, $\widehat{T}_{(0)}$, and $\hat{p}_{(0)}$ and that of the first-order quantities $\hat{v}_{1(1)}$ and $\hat{p}_{(1)}$. Note that the other quantities arising in the equations can be expressed in terms of these quantities. The system can be transformed into the following, which is convenient for the later discussions:

$$\frac{\mathrm{d}\hat{p}_{(0)}}{\mathrm{d}y} = 0, \tag{42a}$$

$$\frac{\mathrm{d}\hat{n}_{(0)}\widehat{w}_{1(1)}}{\mathrm{d}y} = 0 \quad \left(\text{or } \frac{\mathrm{d}\hat{\rho}_{(0)}\hat{v}_{1(1)}}{\mathrm{d}y} = 0\right), \tag{42b}$$

$$\frac{\mathrm{d}}{\mathrm{d}y}\left[\widehat{D}_{\mathrm{AB}}\widehat{T}^{1/2}_{(0)}\left(\frac{\mathrm{d}\chi^{\mathrm{A}}_{(0)}}{\mathrm{d}y} + \frac{k_{\mathrm{T}}}{\widehat{T}_{(0)}}\frac{\mathrm{d}\widehat{T}_{(0)}}{\mathrm{d}y}\right)\right] = \hat{n}_{(0)}\widehat{w}_{1(1)}\frac{\mathrm{d}\chi^{\mathrm{A}}_{(0)}}{\mathrm{d}y}, \tag{42c}$$

$$\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}y}\left(\hat{\mu}\widehat{T}^{1/2}_{(0)}\frac{\mathrm{d}\hat{v}_{2(0)}}{\mathrm{d}y}\right) = \hat{\rho}_{(0)}\hat{v}_{1(1)}\frac{\mathrm{d}\hat{v}_{2(0)}}{\mathrm{d}y}, \tag{42d}$$

$$\frac{\mathrm{d}}{\mathrm{d}y}\left(\hat{\lambda}\widehat{T}^{1/2}_{(0)}\frac{\mathrm{d}\widehat{T}_{(0)}}{\mathrm{d}y}\right) = \frac{5}{2}\hat{n}_{(0)}\widehat{w}_{1(1)}\frac{\mathrm{d}\widehat{T}_{(0)}}{\mathrm{d}y} + \frac{\mathrm{d}}{\mathrm{d}y}k_{\mathrm{T}}\hat{p}_{(0)}\left(\hat{v}^{\mathrm{A}}_{1(1)} - \hat{v}^{\mathrm{B}}_{1(1)}\right) + \frac{\mathrm{d}}{\mathrm{d}y}\left(\hat{\rho}_{(0)}\hat{v}_{1(1)}\hat{v}^2_{2(0)} - \hat{\mu}\widehat{T}^{1/2}_{(0)}\hat{v}_{2(0)}\frac{\mathrm{d}\hat{v}_{2(0)}}{\mathrm{d}y}\right), \tag{42e}$$

$$\hat{v}^{\mathrm{A}}_{1(1)} - \hat{v}^{\mathrm{B}}_{1(1)} = -\frac{\widehat{D}_{\mathrm{AB}}}{\chi^{\mathrm{A}}_{(0)}\chi^{\mathrm{B}}_{(0)}}\frac{\widehat{T}^{1/2}_{(0)}}{\hat{n}_{(0)}}\left(\frac{\mathrm{d}\chi^{A}_{(0)}}{\mathrm{d}y} + \frac{k_{\mathrm{T}}}{\widehat{T}_{(0)}}\frac{\mathrm{d}\widehat{T}_{(0)}}{\mathrm{d}y}\right), \tag{42f}$$

with the boundary conditions at $y = 0$

$$\hat{p}_{(0)} = \hat{p}_{\mathrm{w}} \equiv 1 + \hat{p}^{\mathrm{B}}_{\mathrm{w}}, \tag{43a}$$

$$\chi^{\mathrm{A}}_{(0)} = \chi^{\mathrm{A}}_{\mathrm{w}}, \tag{43b}$$

$$\hat{v}_{2(0)} = 0, \tag{43c}$$

$$\widehat{T}_{(0)} = 1, \tag{43d}$$

and

$$\frac{\mathrm{d}\hat{p}_{(1)}}{\mathrm{d}y} = 0, \tag{44}$$

with the boundary condition at $y = 0$

$$\hat{p}_{(1)} = -a_{\mathrm{V}}\hat{p}_{\mathrm{w}}\widehat{w}_{1(1)} - \left(a_{\mathrm{V}}\widehat{D}_{\mathrm{T}}\frac{\widehat{m}^{\mathrm{B}} - 1}{\widehat{m}_{\mathrm{w}}} + a_{\mathrm{T}}\right)\frac{\mathrm{d}\widehat{T}_{(0)}}{\mathrm{d}y} - \left(a_{\mathrm{V}}\widehat{D}_{\mathrm{AB}}\frac{\widehat{m}^{\mathrm{B}} - 1}{\widehat{m}_{\mathrm{w}}} + a_{\chi}\right)\frac{\mathrm{d}\chi^{\mathrm{A}}_{(0)}}{\mathrm{d}y}. \tag{45}$$

The solution must approach the uniform state at infinity as $y \to \infty$. If we also expand the state at infinity in a power series of ϵ, the conditions at infinity are written as

$$\hat{p}_{(0)} \to \hat{p}_{\infty(0)} \equiv \hat{p}^{\mathrm{A}}_{\infty(0)} + \hat{p}^{\mathrm{B}}_{\infty(0)}, \tag{46a}$$

$$\chi^{\mathrm{A}}_{(0)} \to \chi^{\mathrm{A}}_{\infty(0)} \quad \text{with } \chi^{\mathrm{A}}_{\infty(0)} = \hat{p}^{\mathrm{A}}_{\infty(0)}/\hat{p}_{\infty(0)}, \tag{46b}$$

$$\widehat{T}_{(0)} \to \widehat{T}_{\infty(0)}, \tag{46c}$$

$$\hat{v}_{2(0)} \to \hat{v}_{2\infty(0)}, \tag{46d}$$

$$\widehat{w}_{1(1)} \to \pm 1 \quad (\text{or } \hat{v}_{1(1)} \to \pm 1), \tag{46e}$$

and

$$\hat{p}_{(1)} \to \hat{p}_{\infty(1)} \equiv \hat{p}^{\mathrm{A}}_{\infty(1)} + \hat{p}^{\mathrm{B}}_{\infty(1)}, \tag{47}$$

where

$$\hat{p}_\infty = \hat{p}_{\infty(0)} + \hat{p}_{\infty(1)}\epsilon + \cdots, \qquad \widehat{T}_\infty = \widehat{T}_{\infty(0)} + \widehat{T}_{\infty(1)}\epsilon + \cdots,$$
$$\hat{p}^\alpha_\infty = \hat{p}^\alpha_{\infty(0)} + \hat{p}^\alpha_{\infty(1)}\epsilon + \cdots, \qquad \chi^{\mathrm{A}}_\infty (= p^{\mathrm{A}}_\infty / p_\infty) = \chi^{\mathrm{A}}_{\infty(0)} + \chi^{\mathrm{A}}_{\infty(1)}\epsilon + \cdots,$$
$$\hat{v}_{2\infty} = \hat{v}_{2\infty(0)} + \hat{v}_{2\infty(1)}\epsilon + \cdots,$$

and the $+$ sign is taken for the evaporation and $-$ sign for the condensation in (46e).

Finally, it should be noted that, besides the results listed above, we derived the boundary conditions for $\hat{v}_{2(1)}$, $\hat{p}^{\mathrm{A}}_{(1)}$ and $\widehat{T}_{(1)}$ at $y = 0$ [(37), (39) with $\alpha = \mathrm{A}$, and (40)]. These will be used in the discussion about the evaporation condition in Section 5.2.2.

5. Condensation and evaporation conditions

In this section, we will discuss the evaporation and condensation solutions on the basis of the fluid-dynamic system obtained in the preceding section and derive the relations among the parameters that allow steady evaporation and condensation. We will discuss the evaporation and condensation cases separately, because each case requires a different method. Before starting the detailed discussion, we remark that

$$\hat{p}_{(0)} = \text{const} \ (= \hat{p}_{\mathrm{w}}), \tag{48}$$

$$\hat{n}^\alpha_{(0)}\hat{v}^\alpha_{1(1)} \left(= \hat{n}_{(0)}\chi^\alpha_{(0)}\hat{v}^\alpha_{1(1)}\right) = \text{const}, \qquad \hat{n}_{(0)}\widehat{w}_{1(1)} = \text{const}, \qquad \hat{\rho}_{(0)}\hat{v}_{1(1)} = \text{const}, \tag{49}$$

because of (42a) with (43a), (23a), and (42b). Eq. (48) leads to, with (46a),

$$\hat{p}_{\infty(0)} = \hat{p}_{\mathrm{w}}, \quad \text{i.e.,} \quad \hat{p}_\infty - \hat{p}_{\mathrm{w}} = \mathrm{O}(\epsilon), \tag{50}$$

which means that the pressure of the mixture at infinity cannot be chosen freely from the saturation pressure of the mixture at the interface, irrespective of whether evaporation or condensation takes place.

In the later discussions, instead of (42c)–(42e) themselves, we will use their integrated form:

$$\widehat{D}_{\mathrm{AB}}\widehat{T}^{1/2}_{(0)}\left(\frac{\mathrm{d}\chi^{\mathrm{A}}_{(0)}}{\mathrm{d}y} + \frac{k_{\mathrm{T}}}{\widehat{T}_{(0)}}\frac{\mathrm{d}\widehat{T}_{(0)}}{\mathrm{d}y}\right) = \hat{n}_{(0)}\widehat{w}_{1(1)}\left(\chi^{\mathrm{A}}_{(0)} - \chi_*\right), \tag{51a}$$

$$\frac{1}{2}\hat{\mu}\widehat{T}^{1/2}_{(0)}\frac{\mathrm{d}\hat{v}_{2(0)}}{\mathrm{d}y} = \hat{\rho}_{(0)}\hat{v}_{1(1)}\left(\hat{v}_{2(0)} - v_*\right), \tag{51b}$$

$$\hat{\lambda}\widehat{T}^{1/2}_{(0)}\frac{\mathrm{d}\widehat{T}_{(0)}}{\mathrm{d}y} = \frac{5}{2}\hat{n}_{(0)}\widehat{w}_{1(1)}\left(\widehat{T}_{(0)} - T_*\right) + k_{\mathrm{T}}\hat{p}_{(0)}\left(\hat{v}^{\mathrm{A}}_{1(1)} - \hat{v}^{\mathrm{B}}_{1(1)}\right) + \hat{\rho}_{(0)}\hat{v}_{1(1)}\left(\hat{v}^2_{2(0)} - v^2_*\right) - \hat{\mu}\widehat{T}^{1/2}_{(0)}\hat{v}_{2(0)}\frac{\mathrm{d}\hat{v}_{2(0)}}{\mathrm{d}y}, \tag{51c}$$

where T_*, v_*, and χ_* are arbitrary constants.

5.1. Condensation

We start with considering the flux of Boltzmann's H function for the slowly varying solution.

5.1.1. Flux of Boltzmann's H function and its monotonicity

For the slowly varying solution f^α, we consider the flux of Boltzmann's H function:

$$H_{\mathrm{flux}} = \sum_{\alpha=\mathrm{A,B}} \int \zeta_1 f^\alpha \ln\frac{f^\alpha}{c^\alpha}\,\mathrm{d}\boldsymbol{\zeta},$$

where $c^\alpha = (\widehat{m}^\alpha/\pi)^{3/2}$. If we expand H_{flux} in a power series of ϵ as

$$H_{\mathrm{flux}} = H_{\mathrm{flux}(0)} + H_{\mathrm{flux}(1)}\epsilon + \cdots,$$

the component functions $H_{\mathrm{flux}(m)}$ $(m = 0, 1, 2, \ldots)$ are written as

$$H_{\text{flux}(0)} = \sum_{\alpha=\mathrm{A,B}} \int \zeta_1 f^{\alpha}_{(0)} \ln \frac{f^{\alpha}_{(0)}}{c^{\alpha}} \,\mathrm{d}\boldsymbol{\zeta} = 0, \tag{52}$$

$$H_{\text{flux}(1)} = \sum_{\alpha=\mathrm{A,B}} \int \zeta_1 f^{\alpha}_{(1)} \left(1 + \ln \frac{f^{\alpha}_{(0)}}{c^{\alpha}}\right) \mathrm{d}\boldsymbol{\zeta}, \tag{53}$$

$$H_{\text{flux}(2)} = \sum_{\alpha=\mathrm{A,B}} \int \zeta_1 \left[f^{\alpha}_{(2)} \left(1 + \ln \frac{f^{\alpha}_{(0)}}{c^{\alpha}}\right) + \frac{(f^{\alpha}_{(1)})^2}{2 f^{\alpha}_{(0)}} \right] \mathrm{d}\boldsymbol{\zeta}, \tag{54}$$

and so on. Here $H_{\text{flux}(0)} = 0$ because $f^{\alpha}_{(0)}$ is even in ζ_1. We will first show that $H_{\text{flux}(1)}$ decreases monotonically in y.

Consider the derivative of $H_{\text{flux}(1)}$. With the aid of (17b) and (17c), we have

$$\begin{aligned}
\frac{\mathrm{d}H_{\text{flux}(1)}}{\mathrm{d}y} &= \sum_{\alpha=\mathrm{A,B}} \frac{\mathrm{d}}{\mathrm{d}y} \int \zeta_1 f^{\alpha}_{(1)} \left(1 + \ln \frac{f^{\alpha}_{(0)}}{c^{\alpha}}\right) \mathrm{d}\boldsymbol{\zeta} \\
&= \sum_{\alpha=\mathrm{A,B}} \int \left(1 + \ln \frac{f^{\alpha}_{(0)}}{c^{\alpha}}\right) \zeta_1 \frac{\partial f^{\alpha}_{(1)}}{\partial y} \,\mathrm{d}\boldsymbol{\zeta} + \sum_{\alpha=\mathrm{A,B}} \int \frac{f^{\alpha}_{(1)}}{f^{\alpha}_{(0)}} \zeta_1 \frac{\partial f^{\alpha}_{(0)}}{\partial y} \,\mathrm{d}\boldsymbol{\zeta} \\
&= \sum_{\alpha=\mathrm{A,B}} \int \left(1 + \ln \frac{f^{\alpha}_{(0)}}{c^{\alpha}}\right) \sum_{\beta=\mathrm{A,B}} K^{\beta\alpha} \left[\hat{J}^{\beta\alpha}\left(f^{\beta}_{(0)}, f^{\alpha}_{(2)}\right) + \hat{J}^{\beta\alpha}\left(f^{\beta}_{(2)}, f^{\alpha}_{(0)}\right) + \hat{J}^{\beta\alpha}\left(f^{\beta}_{(1)}, f^{\alpha}_{(1)}\right)\right] \mathrm{d}\boldsymbol{\zeta} \\
&\quad + \sum_{\alpha=\mathrm{A,B}} \int \frac{f^{\alpha}_{(1)}}{f^{\alpha}_{(0)}} \sum_{\beta=\mathrm{A,B}} K^{\beta\alpha} \left[\hat{J}^{\beta\alpha}\left(f^{\beta}_{(0)}, f^{\alpha}_{(1)}\right) + \hat{J}^{\beta\alpha}\left(f^{\beta}_{(1)}, f^{\alpha}_{(0)}\right)\right] \mathrm{d}\boldsymbol{\zeta} \\
&= \sum_{\alpha=\mathrm{A,B}} \int \frac{f^{\alpha}_{(1)}}{f^{\alpha}_{(0)}} \sum_{\beta=\mathrm{A,B}} K^{\beta\alpha} \left[\hat{J}^{\beta\alpha}\left(f^{\beta}_{(0)}, f^{\alpha}_{(1)}\right) + \hat{J}^{\beta\alpha}\left(f^{\beta}_{(1)}, f^{\alpha}_{(0)}\right)\right] \mathrm{d}\boldsymbol{\zeta}.
\end{aligned}$$

Here, in the last equality, it is taken into account that $1 + \ln(f^{\alpha}_{(0)}/c^{\alpha})$ is a collision invariant and that $\hat{J}^{\beta\alpha}$ has the following symmetry property:

$$\begin{aligned}
&\int \psi(\boldsymbol{\zeta}) \hat{J}^{\beta\alpha}(f, g) \,\mathrm{d}\boldsymbol{\zeta} + \int \phi(\boldsymbol{\zeta}) \hat{J}^{\alpha\beta}(g, f) \,\mathrm{d}\boldsymbol{\zeta} \\
&\quad = -\frac{1}{2} \int \left(\psi' + \phi'_* - \psi - \phi_*\right)\left(f'_* g' - f_* g\right) b^{\beta\alpha}\left(\boldsymbol{e} \cdot \widehat{\boldsymbol{V}}/\widehat{V}, \widehat{V}\right) \mathrm{d}\Omega(\boldsymbol{e}) \,\mathrm{d}\boldsymbol{\zeta}_* \,\mathrm{d}\boldsymbol{\zeta},
\end{aligned}$$

with f, g, ϕ, and ψ being arbitrary functions. The proof of this symmetry property is classical and is omitted here. Since the last form can be rewritten in terms of $\mathcal{L}^{\beta\alpha}_{\widehat{T}_{(0)}}$ as

$$\begin{aligned}
&\sum_{\alpha=\mathrm{A,B}} \int \frac{f^{\alpha}_{(1)}}{f^{\alpha}_{(0)}} \sum_{\beta=\mathrm{A,B}} K^{\beta\alpha} \left[\hat{J}^{\beta\alpha}\left(f^{\beta}_{(0)}, f^{\alpha}_{(1)}\right) + \hat{J}^{\beta\alpha}\left(f^{\beta}_{(1)}, f^{\alpha}_{(0)}\right)\right] \mathrm{d}\boldsymbol{\zeta} \\
&\quad = \hat{n}^2_{(0)} \widehat{T}^{1/2}_{(0)} \sum_{\alpha=\mathrm{A,B}} \int \phi^{\alpha} \chi^{\alpha}_{(0)} \left(\sum_{\beta=\mathrm{A,B}} K^{\beta\alpha} \chi^{\beta}_{(0)} \mathcal{L}^{\beta\alpha}_{\widehat{T}_{(0)}}\left(\phi^{\beta}, \phi^{\alpha}\right)\right) E^{\alpha}\left(|\boldsymbol{C}|\right) \mathrm{d}\boldsymbol{C},
\end{aligned}$$

where $\phi^{\alpha} = f^{\alpha}_{(1)}/f^{\alpha}_{(0)}$, the symmetry property (B.10) of $\mathcal{L}^{\beta\alpha}_{\widehat{T}_{(0)}}$ leads to the inequality:

$$\frac{\mathrm{d}H_{\text{flux}(1)}}{\mathrm{d}y} \leqslant 0. \tag{55}$$

Here the equality holds if and only if ϕ^{α} is the collision invariant. Because of (21), the equality condition is equivalent to

$$\frac{\mathrm{d}\chi^{\mathrm{A}}_{(0)}}{\mathrm{d}y} = \frac{\mathrm{d}\widehat{T}_{(0)}}{\mathrm{d}y} = \frac{\mathrm{d}\hat{v}_{2(0)}}{\mathrm{d}y} = 0. \tag{56}$$

Eq. (55) with the condition for equality (56) can be regarded as Boltzmann's H theorem at the first order of ϵ.

Next making use of (55), we will find a monotonic decreasing function of y that is expressed in terms of the quantities occurring in the system (51). Substitution of (18) and (21) into (53) yields the following expression for $H_{\text{flux}(1)}$:

$$H_{\text{flux}(1)} = \hat{n}_{(0)}\widehat{w}_{1(1)}\left(-\frac{3}{2} + \ln\hat{p}_{(0)} - \frac{5}{2}\ln\widehat{T}_{(0)}\right) + \sum_{\alpha=\text{A,B}} \hat{n}^{\alpha}_{(0)}\hat{v}^{\alpha}_{1(1)}\ln\chi^{\alpha}_{(0)}$$
$$+ \frac{1}{\widehat{T}_{(0)}}\left(\hat{\lambda}\widehat{T}^{1/2}_{(0)}\frac{\mathrm{d}\widehat{T}_{(0)}}{\mathrm{d}y} - \hat{p}_{(0)}k_{\mathrm{T}}\left(\hat{v}^{\mathrm{A}}_{1(1)} - \hat{v}^{\mathrm{B}}_{1(1)}\right)\right).$$

The right-hand side can be simplified by substituting (51b) and (51c) with T_* being positive:

$$H_{\text{flux}(1)} = \mathcal{H} + \hat{n}_{(0)}\widehat{w}_{1(1)}\left(1 + \ln\hat{p}_{(0)} - \frac{5}{2}\ln T_*\right), \tag{57}$$

where

$$\mathcal{H} = -\frac{5}{2}\hat{n}_{(0)}\widehat{w}_{1(1)}\left(\frac{T_*}{\widehat{T}_{(0)}} + \ln\frac{\widehat{T}_{(0)}}{T_*}\right) - \hat{\rho}_{(0)}\hat{v}_{1(1)}\frac{(\hat{v}_{2(0)} - v_*)^2}{\widehat{T}_{(0)}} + \sum_{\alpha=\text{A,B}} \hat{n}^{\alpha}_{(0)}\hat{v}^{\alpha}_{1(1)}\ln\chi^{\alpha}_{(0)}. \tag{58}$$

Because of (42a) and (42b), the second term of the right-hand side of (57) is a constant, so that $\mathcal{H}$ satisfies the following inequality because of (55):

$$\frac{\mathrm{d}\mathcal{H}}{\mathrm{d}y} \leqslant 0. \tag{59}$$

Here the equality holds if and only if (56) is satisfied. As will be shown in Section 5.1.2, $\mathcal{H}$ is not only monotonic but also of definite sign. We will make use of both properties in studying the condensation solution in Section 5.1.2.

5.1.2. *Condition for condensation*

Now we will study the condensation solution by the following main three steps [(i)–(iii)] and supplemental two steps [(iv) and (v)]:

(i) The monotonic function $\mathcal{H}$ is positive. This can be shown as follows. Since the flow velocity is common to species at infinity, $\hat{v}_{1(1)}$, $\hat{v}^{\alpha}_{1(1)}$, and $\widehat{w}_{1(1)}$ commonly approach -1 as $y \to \infty$ [see (46e)]. Hence the quantities $\hat{n}_{(0)}\widehat{w}_{1(1)}$, $\hat{\rho}_{(0)}\hat{v}_{1(1)}$, and $\hat{n}^{\alpha}_{(0)}\hat{v}^{\alpha}_{1(1)}$, all of which are constants from (49), are negative. On the other hand, because $1/x + \ln x \geqslant 1$ for $x > 0$ and $\ln x \leqslant 0$ for $0 < x \leqslant 1$, $\ln(\widehat{T}_{(0)}/T_*) + T_*/\widehat{T}_{(0)}$ is positive while $\ln\chi^{\alpha}_{(0)}$ is negative, as far as $\widehat{T}_{(0)} > 0$ and $0 < \chi^{\alpha}_{(0)} < 1$ [note that $T_* > 0$ was assumed just before (57)]. Consequently, $\mathcal{H}$ is positive in the case of condensation.

(ii) Suppose that $\widehat{T}_{(0)} > 0$ and $0 < \chi^{\alpha}_{(0)} < 1$ are assured. Since $\mathcal{H}$ is positive, starting from a certain finite value at $y = 0$, $\mathcal{H}$ monotonically decreases, at most, down to zero as $y \to \infty$. This means that $\mathrm{d}\mathcal{H}/\mathrm{d}y \to 0$ as $y \to \infty$, so that $\mathrm{d}\chi^{\mathrm{A}}_{(0)}/\mathrm{d}y$, $\mathrm{d}\widehat{T}_{(0)}/\mathrm{d}y$, and $\mathrm{d}\hat{v}_{2(0)}/\mathrm{d}y$ all approach zero as $y \to \infty$ because of (56), the condition for equality in (59). As a result, $\chi^{\mathrm{A}}_{(0)}$, $\widehat{T}_{(0)}$, and $\hat{v}_{2(0)}$ approach, respectively, χ_*, T_*, and v_* as $y \to \infty$. This is observed by setting the derivatives to zero in (51) and noting that both $\hat{v}^{\mathrm{A}}_{1(1)}$ and $\hat{v}^{\mathrm{B}}_{1(1)}$ approach -1 as $y \to \infty$. Thus, χ_*, T_*, and v_* are identical with $\chi^{\mathrm{A}}_{\infty(0)}$, $\widehat{T}_{\infty(0)}$, and $\hat{v}_{2\infty(0)}$, respectively.

(iii) In order that $\mathcal{H}$ is positive, it is required that $\widehat{T}_{(0)} > 0$ and $0 < \chi^{\alpha}_{(0)} < 1$. The former is assured because of (43a) and $\hat{p}_{\mathrm{w}} > 0$ from the physical requirement. Satisfying the latter condition is guaranteed if both $\chi^{\mathrm{A}}_{\mathrm{w}}$ and χ_* (or $\chi^{\mathrm{A}}_{\infty(0)}$) are in the interval of (0, 1). It can be shown as follows. Suppose that $\chi^{\mathrm{A}}_{(0)}$ happens to be zero (or 1) at some point, and $0 < \chi^{\mathrm{A}}_{(0)} < 1$ holds up to just before this point in increasing y (remember that $0 < \chi^{\mathrm{A}}_{\mathrm{w}} < 1$). Then the second term in the parentheses of the left-hand side of (51a) vanishes at the point because k_T is zero when $\chi^{\mathrm{A}}_{(0)} = 0$ (or 1).[4] Since $\hat{n}_{(0)}\widehat{w}_{1(1)} < 0$ and $0 < \chi_* < 1$, the right-hand side of (51a) is strictly positive (or negative),

[4] The definition of $\mathcal{D}^{\alpha}$ in (B.6) would not be suitable when $\chi^{\mathrm{A}}_{(0)} = 0$ or 1. We adopted such a function simply because it is conventional. In the case of $\chi^{\mathrm{A}}_{(0)} = 0$ or 1, $\chi^{\mathrm{A}}_{(0)}\chi^{\mathrm{B}}_{(0)}\mathcal{A}^{\mathrm{A}}$ and $\chi^{\mathrm{A}}_{(0)}\chi^{\mathrm{B}}_{(0)}\mathcal{A}^{\mathrm{B}}$ vanish while $\chi^{\mathrm{A}}_{(0)}\chi^{\mathrm{B}}_{(0)}\mathcal{D}^{\mathrm{A}}$ and $\chi^{\mathrm{A}}_{(0)}\chi^{\mathrm{B}}_{(0)}\mathcal{D}^{\mathrm{B}}$ remain finite. Consequently $\widehat{D}_{\mathrm{AB}} > 0$ and

and so is $\mathrm{d}\chi^{\mathrm{A}}_{(0)}/\mathrm{d}y$ because $\widehat{D}_{\mathrm{AB}}$ is positive. Hence $\chi^{\mathrm{A}}_{(0)}$ must be below zero (or above 1) at the left neighboring, which contradicts the assumption.

(iv) If $\chi_* = 0$ (or 1) and $0 < \chi^{\mathrm{A}}_{\mathrm{w}} < 1$, the problem is reduced to that studied recently in [25], in which a condensing vapor flow in the presence of a noncondensable gas is considered. The correspondence comes from the fact that, when $\chi_* = 0$ (or 1), the conservative quantity $\hat{n}^{\mathrm{A}}_{(0)}\hat{v}^{\mathrm{A}}_{1(1)}$ (or $\hat{n}^{\mathrm{B}}_{(0)}\hat{v}^{\mathrm{B}}_{1(1)}$) is zero [see (43b) and (49)], so that $\hat{v}^{\mathrm{A}}_{1(1)} = 0$ at $y = 0$ for the species A (or B) to exist there. In the case, $\chi^{\mathrm{A}}_{(0)}$ varies in the interval of [0, 1]. The function $\mathcal{H}$ can be defined in this interval, and the conclusion of step (ii) remains valid. Further, by the discussion similar to step (iii), the condition $0 \leqslant \chi^{\mathrm{A}}_{(0)} \leqslant 1$ is seen to be assured.

(v) If $\chi^{\mathrm{A}}_{\mathrm{w}} = 0$ (or 1), $\chi^{\mathrm{A}}_{(0)} = 0$ (or $\chi^{\mathrm{B}}_{(0)} = 0$) at $y = 0$, and thus the conservative quantity $\hat{n}^{\mathrm{A}}_{(0)}\hat{v}^{\mathrm{A}}_{1(1)}$ (or $\hat{n}^{\mathrm{B}}_{(0)}\hat{v}^{\mathrm{B}}_{1(1)}$) is again zero. In the case of a condensing flow, $\hat{n}_{(0)}\widehat{w}_{1(1)}$, which is the sum of $\hat{n}^{\mathrm{A}}_{(0)}\hat{v}^{\mathrm{A}}_{1(1)}$ and $\hat{n}^{\mathrm{B}}_{(0)}\hat{v}^{\mathrm{B}}_{1(1)}$, is strictly negative. As a result, χ_* cannot be chosen freely but instead must be equal to $\chi^{\mathrm{A}}_{\mathrm{w}}$. Then, from (51a), $\chi^{\mathrm{A}}_{(0)}$ is shown to be a constant, and the problem is reduced to that of the condensing flow in a one-species system. As in the case of (iv), step (ii) remains valid and the condition $0 \leqslant \chi^{\mathrm{A}}_{(0)} \leqslant 1$ is assured by the discussion similar to step (iii).

In this way, the properties of $\mathcal{H}$, originating from the flux of Boltzmann's H function, assures that $\chi^{\mathrm{A}}_{(0)}$, $\widehat{T}_{(0)}$, and $\hat{v}_{2(0)}$ approach, respectively, $\chi^{\mathrm{A}}_{\infty(0)}$, $\widehat{T}_{\infty(0)}$, and $\hat{v}_{2\infty(0)}$ as $y \to \infty$, as far as $0 \leqslant \chi^{\mathrm{A}}_{\mathrm{w}} \leqslant 1$, $0 \leqslant \chi^{\mathrm{A}}_{\infty(0)} \leqslant 1$, and $\widehat{T}_{\infty(0)} > 0$ are satisfied. The parameters at infinity can be chosen freely relative to those at the surface ($y = 0$), except for the case $\chi^{\mathrm{A}}_{\mathrm{w}} = 0$ or 1. For these singular cases, the problem is reduced to that of a one-species system. In conclusion, as far as $0 < \chi^{\mathrm{A}}_{\mathrm{w}} < 1$ is concerned, there is no restriction on the concentration, the tangential flow velocity, and the temperature in order that steady condensation flow takes place.

We have, however, a restriction on the pressure of the mixture (50):

$$\hat{p}_\infty - \hat{p}_{\mathrm{w}} = \mathrm{O}(\epsilon).$$

To see this restriction closely, we consider the equation and the boundary condition for $\hat{p}_{(1)}$. Because of (23b), $\hat{p}_{(1)}$ is a constant. Thus from the boundary conditions (47) and (41) we have

$$\begin{aligned}(\hat{p}_\infty - \hat{p}_{\mathrm{w}})\epsilon^{-1} &= \hat{p}_{(1)} \\ &= \left[-a_{\mathrm{V}}\hat{p}_{\mathrm{w}}\widehat{w}_{1(1)} - \left(a_{\mathrm{V}}\widehat{D}_{\mathrm{T}}\frac{\widehat{m}^{\mathrm{B}} - 1}{\widehat{m}_{\mathrm{w}}} + a_{\mathrm{T}}\right)\frac{\mathrm{d}\widehat{T}_{(0)}}{\mathrm{d}y} - \left(a_{\mathrm{V}}\widehat{D}_{\mathrm{AB}}\frac{\widehat{m}^{\mathrm{B}} - 1}{\widehat{m}_{\mathrm{w}}} + a_\chi\right)\frac{\mathrm{d}\chi^{\mathrm{A}}_{(0)}}{\mathrm{d}y}\right]_{y=0}. \end{aligned} \tag{60}$$

Here the expansion of $\hat{p}_\infty$ is supposed to terminate at $\mathrm{O}(\epsilon^1)$, i.e.,

$$\hat{p}_\infty = \hat{p}_{\infty(0)} + \hat{p}_{\infty(1)}\epsilon.$$

In the meantime, (51a) and (51c) evaluated at $y = 0$ and $y \to \infty$ yield

$$\left.\frac{\mathrm{d}\widehat{T}_{(0)}}{\mathrm{d}y}\right|_{y=0} = -\frac{\hat{n}_{(0)}\widehat{w}_{1(1)}}{\hat{\lambda}|_{y=0}}\left[\frac{5}{2}\big(\widehat{T}_{\infty(0)} - 1\big) - \frac{k_{\mathrm{T}}|_{y=0}}{\chi^{\mathrm{A}}_{\mathrm{w}}\chi^{\mathrm{B}}_{\mathrm{w}}}\big(\chi^{\mathrm{A}}_{\infty(0)} - \chi^{\mathrm{A}}_{\mathrm{w}}\big) + \frac{\hat{\rho}_{(0)}\hat{v}_{1(1)}}{\hat{n}_{(0)}\widehat{w}_{1(1)}}\hat{v}^2_{2\infty(0)}\right],$$

$$\begin{aligned}\left.\frac{\mathrm{d}\chi^{\mathrm{A}}_{(0)}}{\mathrm{d}y}\right|_{y=0} = \frac{\hat{n}_{(0)}\widehat{w}_{1(1)}}{\widehat{D}_{\mathrm{AB}}|_{y=0}}\Bigg[&\frac{5}{2}\left.\frac{k_{\mathrm{T}}\widehat{D}_{\mathrm{AB}}}{\hat{\lambda}}\right|_{y=0}\big(\widehat{T}_{\infty(0)} - 1\big) + \left.\frac{k_{\mathrm{T}}\widehat{D}_{\mathrm{AB}}}{\hat{\lambda}}\right|_{y=0}\frac{\hat{\rho}_{(0)}\hat{v}_{1(1)}}{\hat{n}_{(0)}\widehat{w}_{1(1)}}\hat{v}^2_{2\infty(0)} \\ &- \left(1 + \frac{1}{\chi^{\mathrm{A}}_{\mathrm{w}}\chi^{\mathrm{B}}_{\mathrm{w}}}\left.\frac{k^2_{\mathrm{T}}\widehat{D}_{\mathrm{AB}}}{\hat{\lambda}}\right|_{y=0}\right)\big(\chi^{\mathrm{A}}_{\infty(0)} - \chi^{\mathrm{A}}_{\mathrm{w}}\big)\Bigg].\end{aligned}$$

Substituting these into (60) yields

$\widehat{D}_{\mathrm{T}} = 0$, and $k_{\mathrm{T}} = 0$ is obtained [see (27a)–(27c)]. Incidentally, the thermal-diffusion factor α_{AB} defined by $\alpha_{\mathrm{AB}} = k_{\mathrm{T}}/\chi^{\mathrm{A}}_{(0)}\chi^{\mathrm{B}}_{(0)}$ is often used in the literature [14,15] in place of k_{T}. This factor does not vanish in general when $\chi^{\mathrm{A}}_{(0)} = 0$ or 1.

$$\begin{aligned}(\hat{p}_\infty - \hat{p}_{\rm w})\epsilon^{-1} = \hat{p}_{(1)} = \bigg[&-a_{\rm V} + \frac{5}{2}\frac{1}{\hat{\lambda}}(a_{\rm T} - k_{\rm T}a_\chi)(\widehat{T}_{\infty(0)} - 1) \\ &+ \bigg(a_{\rm V}\frac{\widehat{m}^{\rm B} - 1}{\widehat{m}_{\rm w}} + \frac{a_\chi}{\widehat{D}_{\rm AB}} - \frac{k_{\rm T}}{\chi_{\rm w}^{\rm A}\chi_{\rm w}^{\rm B}}\frac{1}{\hat{\lambda}}(a_{\rm T} - k_{\rm T}a_\chi)\bigg)(\chi_{\infty(0)}^{\rm A} - \chi_{\rm w}^{\rm A}) \\ &+ \frac{1}{\hat{\lambda}}(a_{\rm T} - k_{\rm T}a_\chi)\frac{\hat{\rho}_{(0)}\hat{v}_{1(1)}}{\hat{n}_{(0)}\widehat{w}_{1(1)}}\hat{v}_{2\infty(0)}^2\bigg]\hat{n}_{(0)}\widehat{w}_{1(1)},\end{aligned}$$

where $k_{\rm T}$, $\hat{\lambda}$, and $\widehat{D}_{\rm AB}$ denote their values at $y = 0$. Finally, expressing the resulting in terms of dimensional quantities, we finally arrive at the relation[5]

$$\begin{aligned}\frac{p_\infty}{p_{\rm w}} = 1 + \bigg[&-a_{\rm V} + \frac{5}{2}\frac{1}{\hat{\lambda}}(a_{\rm T} - k_{\rm T}a_\chi)\bigg(\frac{T_\infty}{T_{\rm w}} - 1\bigg) + \bigg(a_{\rm V}\frac{\widehat{m}^{\rm B} - 1}{\widehat{m}_{\rm w}} + \frac{a_\chi}{\widehat{D}_{\rm AB}} - \frac{k_{\rm T}}{\chi_{\rm w}^{\rm A}\chi_{\rm w}^{\rm B}}\frac{1}{\hat{\lambda}}(a_{\rm T} - k_{\rm T}a_\chi)\bigg)(\chi_\infty^{\rm A} - \chi_{\rm w}^{\rm A}) \\ &+ \frac{1}{\hat{\lambda}}(a_{\rm T} - k_{\rm T}a_\chi)\frac{T_\infty}{T_{\rm w}}\frac{v_{2\infty}^2}{2kT_\infty/m_\infty}\bigg]\frac{T_{\rm w}}{T_\infty}\frac{v_{1\infty}}{c_{\rm w}}.\end{aligned} \tag{61}$$

In the last expression, $\chi_{\infty(0)}^{\rm A}$, $\hat{v}_{2\infty(0)}$, and $\widehat{T}_{\infty(0)}$ are identified with $\chi_\infty^{\rm A}$, $\hat{v}_{2\infty}$, and $\widehat{T}_\infty$, respectively, and m_∞ denotes the average mass of a molecule at a far distance, i.e., $m_\infty = m^{\rm A}\chi_\infty^{\rm A} + m^{\rm B}(1 - \chi_\infty^{\rm A})$. Note that $k_{\rm T}$, $\hat{\lambda}$, and $\widehat{D}_{\rm AB}$ denote their values at $y = 0$. Eq. (61) is the condition for steady condensation.

5.2. *Evaporation*

Our investigation of the condition for the condensation (Section 5.1) essentially relies on the H theorem, the monotonic decrease of the flux of Boltzmann's H function. The theorem holds also for the evaporation case. However it will not give us the benefit in this case, because the function $\mathcal{H} < 0$, so that it is not clear whether $\mathcal{H}$ approaches some constant or grows (negatively) infinitely as $y \to \infty$. Fortunately, we can study the evaporation case more simply by the linear stability analysis of the uniform state at a far distance.

5.2.1. *Linear stability of the uniform state at infinity*

We start with (51) and consider the perturbation of $\chi_{(0)}^{\rm A}$, $\hat{v}_{2(0)}$, and $\widehat{T}_{(0)}$ from the constants χ_*, v_*, and T_*:

$$X = \chi_{(0)}^{\rm A} - \chi_*, \quad Y = \widehat{T}_{(0)} - T_*, \quad Z = \hat{v}_{2(0)} - v_*,$$

and investigate the linear stability of the uniform state solution $\chi_{(0)}^{\rm A} = \chi_*$, $\hat{v}_{2(0)} = v_*$, and $\widehat{T}_{(0)} = T_*$. To do this, we linearize the equations around this state:

$$\begin{aligned}&\widehat{D}_{{\rm AB}*}T_*^{1/2}\bigg(\frac{{\rm d}X}{{\rm d}y} + \frac{k_{{\rm T}*}}{T_*}\frac{{\rm d}Y}{{\rm d}y}\bigg) = \hat{n}_{(0)}\widehat{w}_{1(1)}X, \\ &\frac{1}{2}\hat{\mu}_*T_*^{1/2}\frac{{\rm d}Z}{{\rm d}y} = \hat{\rho}_{(0)}\hat{v}_{1(1)}Z, \\ &\hat{\lambda}_*T_*^{1/2}\frac{{\rm d}Y}{{\rm d}y} = \frac{5}{2}\hat{n}_{(0)}\widehat{w}_{1(1)}Y - \frac{k_{{\rm T}*}\widehat{D}_{{\rm AB}*}}{\chi_*(1-\chi_*)}T_*^{3/2}\bigg(\frac{{\rm d}X}{{\rm d}y} + \frac{k_{{\rm T}*}}{T_*}\frac{{\rm d}T_*}{{\rm d}y}\bigg) + 2\hat{\rho}_{(0)}\hat{v}_{1(1)}v_*Z - \hat{\mu}_*T_*^{1/2}v_*\frac{{\rm d}Z}{{\rm d}y},\end{aligned}$$

where the quantities with subscript $*$ are evaluated with $\chi_{(0)}^{\rm A} = \chi_*$ and $\widehat{T}_{(0)} = T_*$. The linear equations above can be transformed into

$$\begin{bmatrix}\frac{{\rm d}X}{{\rm d}y} \\ \frac{{\rm d}Y}{{\rm d}y} \\ \frac{{\rm d}Z}{{\rm d}y}\end{bmatrix} = \frac{\hat{n}_{(0)}\widehat{w}_{1(1)}}{T_*^{1/2}}M\begin{bmatrix}X \\ Y \\ Z\end{bmatrix}, \tag{62}$$

[5] As has been shown in step (v), the relation (61) holds for $0 < \chi_{\rm w}^{\rm A} < 1$. In the case of $\chi_{\rm w}^{\rm A} = 0$ or 1, $\chi_\infty^{\rm A}$ must be equal to $\chi_{\rm w}^{\rm A}$, and the second line of the equation vanishes; the problem is reduced to that of a one-species system.

where

$$M=\begin{bmatrix}\frac{1}{\widehat{D}_{\mathrm{AB}*}}\left(1+\frac{k_{\mathrm{T}*}^2}{\chi_*(1-\chi_*)}\frac{\widehat{D}_{\mathrm{AB}*}}{\hat{\lambda}_*}\right) & -\frac{5}{2}\frac{k_{\mathrm{T}*}}{T_*\hat{\lambda}_*} & 0\\ -\frac{k_{\mathrm{T}*}T_*}{\chi_*(1-\chi_*)\hat{\lambda}_*} & \frac{5}{2}\frac{1}{\hat{\lambda}_*} & 0\\ 0 & 0 & \frac{2}{\hat{\mu}_*}\frac{\hat{\rho}_{(0)}\hat{v}_{1(1)}}{\hat{n}_{(0)}\widehat{w}_{1(1)}}\end{bmatrix}. \tag{63}$$

Noting that $\widehat{D}_{\mathrm{AB}*}$, $\hat{\mu}_*$, and $\hat{\lambda}_*$ are positive [(27a), (27d), and (27e)], it is easy to see that the real part of all the eigenvalues of this matrix is positive. Therefore, however small the deviation from the uniform state might be, it never vanishes as y grows, since $\hat{n}_{(0)}\widehat{w}_{1(1)}>0$. In other words, it is impossible to reach the uniform state above as $y\to\infty$ starting from any state other than that uniform state. The uniform state can be reached only when the field (at the leading order) is entirely uniform over the half-space, i.e., $\chi_{(0)}^{\mathrm{A}}\equiv\chi_{\mathrm{w}}^{\mathrm{A}}$, $\hat{v}_{2(0)}=0$, and $\widehat{T}_{(0)}\equiv 1$ [see (43b)–(43d)]. Therefore, in addition to the restriction on the pressure

$$\hat{p}_\infty-\hat{p}_{\mathrm{w}}=\mathrm{O}(\epsilon),$$

we have, from (46b)–(46d), the restrictions on the concentration, the tangential flow velocity, and the temperature:

$$\chi_{\infty(0)}^{\mathrm{A}}=\chi_{\mathrm{w}}^{\mathrm{A}},\quad \text{i.e.,}\quad \chi_\infty^{\mathrm{A}}-\chi_{\mathrm{w}}^{\mathrm{A}}=\mathrm{O}(\epsilon),$$
$$\hat{v}_{2\infty(0)}=0,\quad \text{i.e.,}\quad \hat{v}_{2\infty}=\mathrm{O}(\epsilon),$$
$$\widehat{T}_{\infty(0)}=1,\quad \text{i.e.,}\quad \widehat{T}_\infty-1=\mathrm{O}(\epsilon).$$

Incidentally, if the same analysis is applied to the condensation, as is obvious from the discussion above, the uniform state is found to be stable for small perturbations. However, this is not enough to reach the same conclusion as that in Section 5.1.2.

5.2.2. *Condition for evaporation*

Now we will derive the condition for the evaporation explicitly. For the pressure of the mixture, the discussion parallel to that on the condensation case leads to

$$(\hat{p}_\infty-\hat{p}_{\mathrm{w}})\epsilon^{-1}=-a_{\mathrm{V}}\hat{p}_{\mathrm{w}}\widehat{w}_{1(1)}|_{y=0}, \tag{64}$$

because $\widehat{T}_{(0)}$ and $\chi_{(0)}^{\mathrm{A}}$ are constants [see (60)]. As to the concentration, the tangential flow velocity, and the temperature, we need the equations for $\chi_{(1)}^{\mathrm{A}}$, $\hat{v}_{2(1)}$, and $\widehat{T}_{(1)}$. They are obtained by first solving (17c) with $m=2$ for $f_{(2)}^{\alpha}$ and then substituting it into (22) with $m=3$. Since all the quantities at $\mathrm{O}(\epsilon^0)$ are constants, $\hat{v}_{1(1)}$ and $\widehat{w}_{1(1)}$ are also constants [see (49)] and no additional complexity arises in the calculation. The derivation is given in Appendix C. The resulting equations are

$$\frac{\mathrm{d}}{\mathrm{d}y}\left[\widehat{D}_{\mathrm{AB}}\widehat{T}_{(0)}^{1/2}\left(\frac{\mathrm{d}\chi_{(1)}^{\mathrm{A}}}{\mathrm{d}y}+\frac{k_{\mathrm{T}}}{\widehat{T}_{(0)}}\frac{\mathrm{d}\widehat{T}_{(1)}}{\mathrm{d}y}\right)\right]=\hat{n}_{(0)}\widehat{w}_{1(1)}\frac{\mathrm{d}\chi_{(1)}^{\mathrm{A}}}{\mathrm{d}y},$$
$$\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}y}\left(\hat{\mu}\widehat{T}_{(0)}^{1/2}\frac{\mathrm{d}\hat{v}_{2(1)}}{\mathrm{d}y}\right)=\hat{\rho}_{(0)}\hat{v}_{1(1)}\frac{\mathrm{d}\hat{v}_{2(1)}}{\mathrm{d}y},$$
$$\frac{\mathrm{d}}{\mathrm{d}y}\left(\hat{\lambda}\widehat{T}_{(0)}^{1/2}\frac{\mathrm{d}\widehat{T}_{(1)}}{\mathrm{d}y}-k_{\mathrm{T}}\hat{p}_{(0)}\left(\hat{v}_{1(2)}^{\mathrm{A}}-\hat{v}_{1(2)}^{\mathrm{B}}\right)\right)$$
$$=\frac{5}{2}\hat{n}_{(0)}\widehat{w}_{1(1)}\frac{\mathrm{d}\widehat{T}_{(1)}}{\mathrm{d}y}+\frac{\mathrm{d}}{\mathrm{d}y}\left(2\hat{\rho}_{(0)}\hat{v}_{1(1)}\hat{v}_{2(0)}\hat{v}_{2(1)}-\hat{\mu}\widehat{T}_{(0)}^{1/2}\hat{v}_{2(0)}\frac{\mathrm{d}\hat{v}_{2(1)}}{\mathrm{d}y}\right),$$

which are linear equations for $\chi_{(1)}^{\mathrm{A}}$, $\hat{v}_{2(1)}$, and $\widehat{T}_{(1)}$. Note that except them the quantities in the equations are constants. The stability analysis of this linear system is parallel to that in the preceding section and concludes

$$\widehat{T}_{(1)}=\text{const},\quad \chi_{(1)}^{\mathrm{A}}=\text{const},\quad \hat{v}_{2(1)}=\text{const},$$

and thus

$$\hat{p}_{(1)}^{\mathrm{A}}=\chi_{(0)}^{\mathrm{A}}\hat{p}_{(1)}+\chi_{(1)}^{\mathrm{A}}\hat{p}_{(0)}=\text{const}. \tag{65}$$

Therefore, from (37), (39) with $\alpha = \mathrm{A}$, and (40), we obtain

$$\hat{v}_{2\infty(1)} = \hat{v}_{2\infty}\epsilon^{-1} = \hat{v}_{2(1)} = \hat{v}_{2(1)}|_{y=0} = 0, \tag{66}$$

$$\hat{p}^{\mathrm{A}}_{\infty(1)} = \left(\hat{p}^{\mathrm{A}}_{\infty} - 1\right)\epsilon^{-1} = \hat{p}^{\mathrm{A}}_{(1)} = -a^{\mathrm{A}}_{\mathrm{V}}\widehat{w}_{1(1)}|_{y=0}, \tag{67}$$

$$\widehat{T}_{\infty(1)} = \left(\widehat{T}_{\infty} - 1\right)\epsilon^{-1} = \widehat{T}_{(1)} = -c_{\mathrm{V}}\widehat{w}_{1(1)}|_{y=0}. \tag{68}$$

Here the expansions of $\hat{p}^{\mathrm{A}}_{\infty}$, $\hat{v}_{2\infty}$, and $\widehat{T}_{\infty}$ are supposed to terminate at $\mathrm{O}(\epsilon^1)$. Finally, expressing (64) and (66)–(68) in terms of dimensional quantities, we arrive at the relations:

$$\frac{p_\infty}{p_{\mathrm{w}}} = 1 - a_{\mathrm{V}}\frac{v_{1\infty}}{c_{\mathrm{w}}}, \tag{69}$$

$$\frac{p^{\mathrm{A}}_\infty}{p^{\mathrm{A}}_{\mathrm{w}}} = 1 - a^{\mathrm{A}}_{\mathrm{V}}\frac{v_{1\infty}}{c_{\mathrm{w}}}, \tag{70}$$

$$\frac{T_\infty}{T_{\mathrm{w}}} = 1 - c_{\mathrm{V}}\frac{v_{1\infty}}{c_{\mathrm{w}}}, \tag{71}$$

$$v_{2\infty} = 0. \tag{72}$$

The last equation means that evaporation always takes place perpendicularly to the surface. Eqs. (69)–(72) are the set of conditions for steady evaporation.

5.3. Summary

In Section 5, we derived the conditions in order that evaporation or condensation takes place. The result shows that there is a qualitative difference between the evaporation and condensation cases. For the former, there are four conditions (69)–(72), whereas there is only one condition (61) for the latter. This is a natural extension of the existing result for a single-species vapor to a mixture of vapors in the sense that the single condition for the condensation has a dependence on the concentration and that one additional condition on the concentration is required besides the conditions for the pressure, the tangential flow velocity, and the temperature in the case of evaporation.

6. Concluding remarks

In the present paper, we have considered the half-space problem of evaporation and condensation of a binary mixture of vapors. Assuming that the Mach number of the perpendicular component of the flow is small, we considered the solution that varies slowly in the scale of the mean free path and derived the fluid-dynamic system that describes the behavior of the solution by a formal but systematic asymptotic analysis. Based on that system, we studied the behavior of the slowly varying solution in a rather indirect way and derived the conditions that must be satisfied in order that steady evaporation or condensation takes place. The conditions relate the parameters characterizing the state of the condensed phase to those characterizing the state of the mixture at a far distance.

Our discussion relies on the H theorem, the monotonic decrease of the flux of Boltzmann's H function, in the case of condensation and on the linear stability analysis in the case of evaporation. The resulting conditions are qualitatively different between the evaporation and the condensation cases: there is a single condition for the condensation, while there are four conditions for the evaporation. This is a natural extension of the existing result [7,1] for a single-species vapor to a mixture of vapors in the sense that the single condition for the condensation has a dependence on the concentration and that one additional condition on the concentration is required besides the conditions for the pressure, the tangential flow velocity, and the temperature in the case of the evaporation. The present result supports the assumption that was made in performing the numerical computation of evaporating flow in the literature (e.g. [26,27]) in the regime of small Mach number.

In the present paper, we did not specify the model of intermolecular potential but rather kept it arbitrary as long as the collision frequency and the transport coefficients can be defined properly [see the end of Section 3.2 and (28)]. In this sense, the present work may be considered as the generalization of [12] that was limited to the BGK-type model Boltzmann system, such as the models proposed in [9–11]. In [12], only the case of $v_{2\infty} = 0$ was investigated, and, thanks to the monotonic behavior of the concentration and the temperature themselves for those models, a more direct solution approach was taken to arrive at the conditions (61) and (69)–(71) with $v_{2\infty} = 0$. Incidentally, the direct approach in [12] can be extended to the case of $v_{2\infty} \neq 0$, as far as the BGK-type models are concerned. For this case, although the temperature is no longer monotonic, the concentration and the tangential flow velocity remain monotonic, so that the explicit parametric expression for the temperature in terms of the tangential flow velocity can be obtained. The reader is referred to [25] for the example of this type of analysis though a similar but different physical problem is studied for hard sphere gases there. That parametric expression enables us to directly arrive at the same conclusion as the present paper, i.e., the conditions (61) and (69)–(72).

As is mentioned in Section 1, the objective of the present work is to generalize the previous contribution [12] of the first author by overcoming the difficulty arising from the generally nonmonotonic behavior of the fluid-dynamic quantities of the slowly varying solution in the case of the Boltzmann equation for mixtures, when trying to reach the conditions (61) and (69)–(72). The reader who is interested in the quantitative performance of the simplified kinetic models is referred to Fig. 7 in [28], where numerical simulations of the two-surface problem are carried out by using both the Garzó–Santos–Brey BGK-type model [9] and the hard-sphere Boltzmann equation and they agree well with each other. However, this example does not cover the case where the fluid-dynamic quantities are nonmonotonic. More detailed examination would be required to draw a rather general conclusion on the quantitative performance. It should also be noted that the conditions (61) and (69)–(72) are quantitatively affected by the choice of the molecular model. For instance, the comparison between the results in [23] and [21] shows that a_{V} and c_{V} could be different, respectively, by $3 \sim 7$ and 8% between the hard-sphere and Maxwell molecular models.

Finally, we briefly mention the influence of the generalization of the kinetic boundary condition. In the present work, the conditions (61) and (69)–(72) for the condensation and evaporation are derived on the basis of the kinetic boundary condition (3a) [or (12a)] assuming the perfect accommodation of the molecules coming to the interface from the gas phase (see the end of Section 2). If the accommodation is only partial, (29) still holds but linear functionals of $f^{\alpha}_{\mathrm{K}(1)}$ (or Φ^{α}) and $f^{\alpha}_{\mathrm{S}(0)}$ also appear on the right-hand sides of (32), (34b), and (35b), as far as the accommodation rate is not too small ($\gg \epsilon$). However, this difference does not affect the structure of the relations (36)–(38) and merely changes the values of the coefficients a^{α}_{V}, c_{V}, etc. when the boundary is *locally isotropic*. The reader is referred to Section 3.4 in [1] for the detailed discussion on this issue. Therefore, the conditions (61) and (69)–(72) remains unchanged for more general kinetic boundary conditions discussed in [3,1,2], including that the evaporation takes place only perpendicularly. The mathematical proof on the problem (35) with a linear functional of Φ^{α} on the right-hand side of (35b) is required to make this statement rigorous. In the case of a single-species vapor, a mathematical proof for such generalization is given in [29].

Acknowledgements

The first author (S.T.) thanks Professor Kazuo Aoki for his interest in this work. The essential part of this work was done during the stay of S.T. at the U. F. R. de Mathématiques, Université Denis Diderot by the scholastic exchange agreement between Kyoto University and Université Denis Diderot. The present work is supported by the Grants-in-Aid for Scientific Research (Nos. 14350047 and 15760108) from the Japan Society for the Promotion of Science and by the COE program of the Japanese Ministry of Education, Culture, Sports, Science and Technology for Research and Education on Complex Functional Mechanical Systems.

Appendix A. Component functions of macroscopic quantities

Here we summarize the expressions of the component functions of $\hat{n}^{\alpha}$, $\hat{p}^{\alpha}$, $\widehat{T}^{\alpha}$, $\hat{v}^{\alpha}_1$, etc. in terms of the component functions of f^{α}. In the derivation, the assumption $\hat{v}^{\alpha}_{1(0)} = \hat{v}_{1(0)} = \widehat{w}_{1(0)} = 0$ was used. Einstein's summation convention is suppressed in this appendix.

A.1. Component functions of $\mathrm{O}(\epsilon^0)$*-quantities*

$$\hat{n}^{\alpha}_{(0)} = \int f^{\alpha}_{(0)}\,\mathrm{d}\boldsymbol{\zeta}, \quad \hat{\rho}^{\alpha}_{(0)} = \widehat{m}^{\alpha}\hat{n}^{\alpha}_{(0)}, \quad \hat{v}^{\alpha}_{2(0)} = \frac{1}{\hat{n}^{\alpha}_{(0)}}\int \zeta_2 f^{\alpha}_{(0)}\,\mathrm{d}\boldsymbol{\zeta},$$

$$\hat{p}^{\alpha}_{(0)} = \hat{n}^{\alpha}_{(0)}\widehat{T}^{\alpha}_{(0)} = \frac{2}{3}\int \left|\boldsymbol{\zeta} - \hat{\boldsymbol{v}}^{\alpha}_{(0)}\right|^2 \widehat{m}^{\alpha} f^{\alpha}_{(0)}\,\mathrm{d}\boldsymbol{\zeta},$$

$$\hat{p}^{\alpha}_{ii(0)} = 2\int \zeta_i^2 \widehat{m}^{\alpha} f^{\alpha}_{(0)}\,\mathrm{d}\boldsymbol{\zeta} \quad (i=1,3),$$
$$\hat{p}^{\alpha}_{22(0)} = 2\int \big(\zeta_2 - \hat{v}^{\alpha}_{2(0)}\big)^2 \widehat{m}^{\alpha} f^{\alpha}_{(0)}\,\mathrm{d}\boldsymbol{\zeta},$$
$$\hat{p}^{\alpha}_{21(0)} = \hat{p}^{\alpha}_{12(0)} = 2\int \zeta_1\big(\zeta_2 - \hat{v}^{\alpha}_{2(0)}\big) \widehat{m}^{\alpha} f^{\alpha}_{(0)}\,\mathrm{d}\boldsymbol{\zeta},$$
$$\hat{q}^{\alpha}_{1(0)} = \int \zeta_1\big|\boldsymbol{\zeta} - \hat{\boldsymbol{v}}^{\alpha}_{(0)}\big|^2 \widehat{m}^{\alpha} f^{\alpha}_{(0)}\,\mathrm{d}\boldsymbol{\zeta},$$

and

$$\hat{n}_{(0)} = \hat{n}^{\mathrm{A}}_{(0)} + \hat{n}^{\mathrm{B}}_{(0)}, \quad \hat{\rho}_{(0)} = \hat{\rho}^{\mathrm{A}}_{(0)} + \hat{\rho}^{\mathrm{B}}_{(0)}, \quad \hat{v}_{2(0)} = \frac{1}{\hat{\rho}_{(0)}}\big(\hat{\rho}^{\mathrm{A}}_{(0)}\hat{v}^{\mathrm{A}}_{2(0)} + \hat{\rho}^{\mathrm{B}}_{(0)}\hat{v}^{\mathrm{B}}_{2(0)}\big),$$
$$\hat{p}_{(0)} = \hat{n}_{(0)}\widehat{T}_{(0)} = \sum_{\alpha=\mathrm{A,B}}\left(\hat{p}^{\alpha}_{(0)} + \frac{2}{3}\hat{\rho}^{\alpha}_{(0)}\big(V^{\alpha}_{2(0)}\big)^2\right),$$
$$\hat{p}_{ii(0)} = \sum_{\alpha=\mathrm{A,B}} \hat{p}^{\alpha}_{ii(0)} \quad (i=1,3), \qquad \hat{p}_{22(0)} = \sum_{\alpha=\mathrm{A,B}}\big(\hat{p}^{\alpha}_{22(0)} + 2\hat{\rho}^{\alpha}_{(0)}\big(V^{\alpha}_{2(0)}\big)^2\big),$$
$$\hat{p}_{21(0)} = \hat{p}_{12(0)} = \sum_{\alpha=\mathrm{A,B}} \hat{p}^{\alpha}_{21(0)}, \qquad \hat{q}_{1(0)} = \sum_{\alpha=\mathrm{A,B}}\big(\hat{q}^{\alpha}_{1(0)} + \hat{p}^{\alpha}_{12(0)}V^{\alpha}_{2(0)}\big),$$
$$\chi^{\alpha}_{(0)} = \hat{n}^{\alpha}_{(0)}/\hat{n}_{(0)},$$

where $\hat{\boldsymbol{v}}^{\alpha}_{(0)} = (0, \hat{v}^{\alpha}_{2(0)}, 0)$ and $V^{\alpha}_{2(0)} = \hat{v}^{\alpha}_{2(0)} - \hat{v}_{2(0)}$.

A.2. *Component functions of* $\mathrm{O}(\epsilon^1)$*-quantities*

$$\hat{n}^{\alpha}_{(1)} = \int f^{\alpha}_{(1)}\,\mathrm{d}\boldsymbol{\zeta}, \quad \hat{\rho}^{\alpha}_{(1)} = \widehat{m}^{\alpha}\hat{n}^{\alpha}_{(1)}, \quad \hat{v}^{\alpha}_{1(1)} = \frac{1}{\hat{n}^{\alpha}_{(0)}}\int \zeta_1 f^{\alpha}_{(1)}\,\mathrm{d}\boldsymbol{\zeta},$$
$$\hat{v}^{\alpha}_{2(1)} = \frac{1}{\hat{n}^{\alpha}_{(0)}}\left(\int \zeta_2 f^{\alpha}_{(1)}\,\mathrm{d}\boldsymbol{\zeta} - \hat{n}^{\alpha}_{(1)}\hat{v}^{\alpha}_{2(0)}\right),$$
$$\hat{p}^{\alpha}_{(1)} = \hat{n}^{\alpha}_{(1)}\widehat{T}^{\alpha}_{(0)} + \hat{n}^{\alpha}_{(0)}\widehat{T}^{\alpha}_{(1)} = \frac{2}{3}\int \big|\boldsymbol{\zeta} - \hat{\boldsymbol{v}}^{\alpha}_{(0)}\big|^2 \widehat{m}^{\alpha} f^{\alpha}_{(1)}\,\mathrm{d}\boldsymbol{\zeta},$$
$$\hat{p}^{\alpha}_{ii(1)} = 2\int \zeta_i^2 \widehat{m}^{\alpha} f^{\alpha}_{(1)}\,\mathrm{d}\boldsymbol{\zeta} \quad (i=1,3),$$
$$\hat{p}^{\alpha}_{22(1)} = 2\int \big(\zeta_2 - \hat{v}^{\alpha}_{2(0)}\big)^2 \widehat{m}^{\alpha} f^{\alpha}_{(1)}\,\mathrm{d}\boldsymbol{\zeta},$$
$$\hat{p}^{\alpha}_{21(1)} = \hat{p}^{\alpha}_{12(1)} = 2\int \zeta_1\big(\zeta_2 - \hat{v}^{\alpha}_{2(0)}\big) \widehat{m}^{\alpha} f^{\alpha}_{(1)}\,\mathrm{d}\boldsymbol{\zeta},$$
$$\hat{q}^{\alpha}_{1(1)} = \int \zeta_1\big|\boldsymbol{\zeta} - \hat{\boldsymbol{v}}^{\alpha}_{(0)}\big|^2 \widehat{m}^{\alpha} f^{\alpha}_{(1)}\,\mathrm{d}\boldsymbol{\zeta} - \frac{3}{2}\hat{p}^{\alpha}_{(0)}\hat{v}^{\alpha}_{1(1)} - \hat{p}^{\alpha}_{11(0)}\hat{v}^{\alpha}_{1(1)} - \hat{p}^{\alpha}_{12(0)}\hat{v}^{\alpha}_{2(1)},$$

and

$$\hat{n}_{(1)} = \hat{n}^{\mathrm{A}}_{(1)} + \hat{n}^{\mathrm{B}}_{(1)}, \quad \hat{\rho}_{(1)} = \hat{\rho}^{\mathrm{A}}_{(1)} + \hat{\rho}^{\mathrm{B}}_{(1)}, \quad \hat{v}_{1(1)} = \frac{1}{\hat{\rho}_{(0)}}\big(\hat{\rho}^{\mathrm{A}}_{(0)}\hat{v}^{\mathrm{A}}_{1(1)} + \hat{\rho}^{\mathrm{B}}_{(0)}\hat{v}^{\mathrm{B}}_{1(1)}\big),$$
$$\hat{v}_{2(1)} = \frac{1}{\hat{\rho}_{(0)}}\big(\hat{\rho}^{\mathrm{A}}_{(0)}\hat{v}^{\mathrm{A}}_{2(1)} + \hat{\rho}^{\mathrm{B}}_{(0)}\hat{v}^{\mathrm{B}}_{2(1)} + \hat{\rho}^{\mathrm{A}}_{(1)}V^{\mathrm{A}}_{2(0)} + \hat{\rho}^{\mathrm{B}}_{(1)}V^{\mathrm{B}}_{2(0)}\big),$$
$$\hat{p}_{(1)} = \hat{n}_{(1)}\widehat{T}_{(0)} + \hat{n}_{(0)}\widehat{T}_{(1)} = \sum_{\alpha=\mathrm{A,B}}\left(\hat{p}^{\alpha}_{(1)} + \frac{2}{3}\hat{\rho}^{\alpha}_{(1)}\big(V^{\alpha}_{2(0)}\big)^2 + \frac{4}{3}\hat{\rho}^{\alpha}_{(0)}V^{\alpha}_{2(0)}V^{\alpha}_{2(1)}\right),$$

$$\hat{p}_{ii(1)} = \sum_{\alpha=\mathrm{A,B}} \hat{p}^{\alpha}_{ii(1)} \quad (i = 1, 3),$$

$$\hat{p}_{22(1)} = \sum_{\alpha=\mathrm{A,B}} \bigl(\hat{p}^{\alpha}_{22(1)} + 2\hat{\rho}^{\alpha}_{(1)}\bigl(V^{\alpha}_{2(0)}\bigr)^2 + 4\hat{\rho}^{\alpha}_{(0)} V^{\alpha}_{2(0)} V^{\alpha}_{2(1)}\bigr),$$

$$\hat{p}_{21(1)} = \hat{p}_{12(1)} = \sum_{\alpha=\mathrm{A,B}} \bigl(\hat{p}^{\alpha}_{21(1)} + 2\hat{\rho}^{\alpha}_{(0)} V^{\alpha}_{1(1)} V^{\alpha}_{2(0)}\bigr),$$

$$\hat{q}_{1(1)} = \sum_{\alpha=\mathrm{A,B}} \left(\hat{q}^{\alpha}_{1(1)} + \hat{p}^{\alpha}_{12(1)} V^{\alpha}_{2(0)} + \hat{p}^{\alpha}_{11(0)} V^{\alpha}_{1(1)} + \hat{p}^{\alpha}_{12(0)} V^{\alpha}_{2(1)} + \frac{3}{2}\hat{p}^{\alpha}_{(0)} V^{\alpha}_{1(1)} + \hat{\rho}^{\alpha}_{(0)} V^{\alpha}_{1(1)}\bigl(V^{\alpha}_{2(0)}\bigr)^2\right),$$

$$\chi^{\alpha}_{(1)} = \bigl(n^{\alpha}_{(1)} - \chi^{\alpha}_{(0)} n_{(1)}\bigr)/n_{(0)}, \qquad w_{1(1)} = \chi^{\mathrm{A}}_{(0)} v^{\mathrm{A}}_{1(1)} + \chi^{\mathrm{B}}_{(0)} v^{\mathrm{B}}_{1(1)},$$

where $V^{\alpha}_{1(1)} = \hat{v}^{\alpha}_{1(1)} - \hat{v}_{1(1)}$ and $V^{\alpha}_{2(1)} = \hat{v}^{\alpha}_{2(1)} - \hat{v}_{2(1)}$.

A.3. Component functions of $\mathrm{O}(\epsilon^2)$*-quantities*

$$\hat{n}^{\alpha}_{(2)} = \int f^{\alpha}_{(2)}\,\mathrm{d}\boldsymbol{\zeta}, \quad \hat{\rho}^{\alpha}_{(2)} = \widehat{m}^{\alpha}\hat{n}^{\alpha}_{(2)}, \quad \hat{v}^{\alpha}_{1(2)} = \frac{1}{\hat{n}^{\alpha}_{(0)}}\left(\int \zeta_1 f^{\alpha}\,\mathrm{d}\boldsymbol{\zeta} - \hat{n}^{\alpha}_{(1)}\hat{v}^{\alpha}_{1(1)}\right),$$

$$\hat{v}^{\alpha}_{2(2)} = \frac{1}{\hat{n}^{\alpha}_{(0)}}\left(\int \zeta_2 f^{\alpha}\,\mathrm{d}\boldsymbol{\zeta} - \hat{n}^{\alpha}_{(1)}\hat{v}^{\alpha}_{2(1)} - \hat{n}^{\alpha}_{(2)}\hat{v}^{\alpha}_{2(0)}\right),$$

$$\hat{p}^{\alpha}_{(2)} = \hat{n}^{\alpha}_{(0)}\widehat{T}^{\alpha}_{(2)} + \hat{n}^{\alpha}_{(1)}\widehat{T}^{\alpha}_{(1)} + \hat{n}^{\alpha}_{(2)}\widehat{T}^{\alpha}_{(0)} = \frac{2}{3}\int \bigl|\boldsymbol{\zeta} - \hat{\boldsymbol{v}}^{\alpha}_{(0)}\bigr|^2 \widehat{m}^{\alpha} f^{\alpha}_{(2)}\,\mathrm{d}\boldsymbol{\zeta} - \frac{2}{3}\hat{\rho}^{\alpha}_{(0)}\bigl|\hat{\boldsymbol{v}}^{\alpha}_{(1)}\bigr|^2,$$

$$\hat{p}^{\alpha}_{11(2)} = 2\int \zeta_1^2 \widehat{m}^{\alpha} f^{\alpha}_{(2)}\,\mathrm{d}\boldsymbol{\zeta} - 2\hat{\rho}^{\alpha}_{(0)}\bigl(\hat{v}^{\alpha}_{1(1)}\bigr)^2,$$

$$\hat{p}^{\alpha}_{22(2)} = 2\int \bigl(\zeta_2 - \hat{v}^{\alpha}_{2(0)}\bigr)^2 \widehat{m}^{\alpha} f^{\alpha}_{(2)}\,\mathrm{d}\boldsymbol{\zeta} - 2\hat{\rho}^{\alpha}_{(0)}\bigl(\hat{v}^{\alpha}_{2(1)}\bigr)^2,$$

$$\hat{p}^{\alpha}_{33(2)} = 2\int \zeta_3^2 \widehat{m}^{\alpha} f^{\alpha}_{(2)}\,\mathrm{d}\boldsymbol{\zeta},$$

$$\hat{p}^{\alpha}_{21(2)} = \hat{p}^{\alpha}_{12(2)} = 2\int \zeta_1\bigl(\zeta_2 - \hat{v}^{\alpha}_{2(0)}\bigr)\widehat{m}^{\alpha} f^{\alpha}_{(2)}\,\mathrm{d}\boldsymbol{\zeta} - 2\hat{\rho}^{\alpha}_{(0)}\hat{v}^{\alpha}_{1(1)}\hat{v}^{\alpha}_{2(1)},$$

$$\begin{aligned}\hat{q}^{\alpha}_{1(2)} = {} & \int \zeta_1\bigl|\boldsymbol{\zeta} - \hat{\boldsymbol{v}}^{\alpha}_{(0)}\bigr|^2 \widehat{m}^{\alpha} f^{\alpha}_{(2)}\,\mathrm{d}\boldsymbol{\zeta} - \frac{3}{2}\hat{p}^{\alpha}_{(1)}\hat{v}^{\alpha}_{1(1)} - \frac{3}{2}\hat{p}^{\alpha}_{(0)}\hat{v}^{\alpha}_{1(2)} \\ & - \hat{p}^{\alpha}_{11(1)}\hat{v}^{\alpha}_{1(1)} - \hat{p}^{\alpha}_{12(1)}\hat{v}^{\alpha}_{2(1)} - \hat{p}^{\alpha}_{11(0)}\hat{v}^{\alpha}_{1(2)} - \hat{p}^{\alpha}_{12(0)}\hat{v}^{\alpha}_{2(2)}\end{aligned}$$

and

$$\hat{n}_{(2)} = \hat{n}^{\mathrm{A}}_{(2)} + \hat{n}^{\mathrm{B}}_{(2)}, \qquad \hat{\rho}_{(2)} = \hat{\rho}^{\mathrm{A}}_{(2)} + \hat{\rho}^{\mathrm{B}}_{(2)},$$

$$\hat{v}_{1(2)} = \frac{1}{\hat{\rho}_{(0)}}\bigl(\hat{\rho}^{\mathrm{A}}_{(0)}\hat{v}^{\mathrm{A}}_{1(2)} + \hat{\rho}^{\mathrm{B}}_{(0)}\hat{v}^{\mathrm{B}}_{1(2)} + \hat{\rho}^{\mathrm{A}}_{(1)} V^{\mathrm{A}}_{1(1)} + \hat{\rho}^{\mathrm{B}}_{(1)} V^{\mathrm{B}}_{1(1)}\bigr),$$

$$\hat{v}_{2(2)} = \frac{1}{\hat{\rho}_{(0)}}\bigl(\hat{\rho}^{\mathrm{A}}_{(0)}\hat{v}^{\mathrm{A}}_{2(2)} + \hat{\rho}^{\mathrm{B}}_{(0)}\hat{v}^{\mathrm{B}}_{2(2)} + \hat{\rho}^{\mathrm{A}}_{(1)} V^{\mathrm{A}}_{2(1)} + \hat{\rho}^{\mathrm{B}}_{(1)} V^{\mathrm{B}}_{2(1)} + \hat{\rho}^{\mathrm{A}}_{(2)} V^{\mathrm{A}}_{2(0)} + \hat{\rho}^{\mathrm{B}}_{(2)} V^{\mathrm{B}}_{2(0)}\bigr),$$

$$\hat{p}_{12(2)} = \hat{p}_{21(2)} = \sum_{\alpha=\mathrm{A,B}} \bigl(\hat{p}^{\alpha}_{12(2)} + 2\hat{\rho}^{\alpha}_{(1)} V^{\alpha}_{1(1)} V^{\alpha}_{2(0)} + 2\hat{\rho}^{\alpha}_{(0)}\bigl(V^{\alpha}_{1(1)} V^{\alpha}_{2(1)} + V^{\alpha}_{1(2)} V^{\alpha}_{2(0)}\bigr)\bigr),$$

$$\begin{aligned}\hat{q}_{1(2)} = \sum_{\alpha=\mathrm{A,B}} \Bigl(& \hat{q}^{\alpha}_{1(2)} + \hat{p}^{\alpha}_{11(1)} V^{\alpha}_{1(1)} + \hat{p}^{\alpha}_{11(0)} V^{\alpha}_{1(2)} \\ & + \hat{p}^{\alpha}_{12(2)} V^{\alpha}_{2(0)} + \hat{p}^{\alpha}_{12(1)} V^{\alpha}_{2(1)} + \hat{p}^{\alpha}_{12(0)} V^{\alpha}_{2(2)} + \frac{3}{2}\hat{p}^{\alpha}_{(1)} V^{\alpha}_{1(1)} + \frac{3}{2}\hat{p}^{\alpha}_{(0)} V^{\alpha}_{1(2)}\end{aligned}$$

$$+\hat{\rho}^{\alpha}_{(1)}V^{\alpha}_{1(1)}\big(V^{\alpha}_{2(0)}\big)^2+\hat{\rho}^{\alpha}_{(0)}\big(V^{\alpha}_{1(2)}\big(V^{\alpha}_{2(0)}\big)^2+2V^{\alpha}_{1(1)}V^{\alpha}_{2(0)}V^{\alpha}_{2(1)}\big)\bigg),$$

$$\widehat{w}_{1(2)}=\chi^{\mathrm{A}}_{(0)}\hat{v}^{\mathrm{A}}_{1(2)}+\chi^{\mathrm{B}}_{(0)}\hat{v}^{\mathrm{B}}_{1(2)}+\chi^{\mathrm{A}}_{(1)}\hat{v}^{\mathrm{A}}_{1(1)}+\chi^{\mathrm{B}}_{(1)}\hat{v}^{\mathrm{B}}_{1(1)},$$

where $\hat{\boldsymbol{v}}^{\alpha}_{(1)}=(\hat{v}^{\alpha}_{1(1)},\hat{v}^{\alpha}_{2(1)},0)$, $V^{\alpha}_{1(2)}=\hat{v}^{\alpha}_{1(2)}-\hat{v}_{1(2)}$, and $V^{\alpha}_{2(2)}=\hat{v}^{\alpha}_{2(2)}-\hat{v}_{2(2)}$.

Appendix B. Component function $f^{\alpha}_{(1)}$ and some properties of the functions $\mathcal{A}^{\alpha}$, $\mathcal{B}^{\alpha}$, and $\mathcal{D}^{\alpha}$

In this appendix, we solve (17b) and derive some important properties of the functions related to the transport coefficients.

B.1. Component function $f^{\alpha}_{(1)}$

Substituting (18) into the right-hand side of (17b) and taking account of (20) yield

$$\sum_{\beta=\mathrm{A,B}}K^{\beta\alpha}\big[\hat{J}^{\beta\alpha}\big(f^{\beta}_{(1)},f^{\alpha}_{(0)}\big)+\hat{J}^{\beta\alpha}\big(f^{\beta}_{(0)},f^{\alpha}_{(1)}\big)\big]$$
$$=\zeta_1 f^{\alpha}_{(0)}\bigg[\frac{1}{\chi^{\alpha}_{(0)}}\frac{\mathrm{d}\chi^{\alpha}_{(0)}}{\mathrm{d}y}+\frac{2\widehat{m}^{\alpha}(\zeta_2-\hat{v}_{2(0)})}{\widehat{T}_{(0)}}\frac{\mathrm{d}\hat{v}_{2(0)}}{\mathrm{d}y}+\bigg(\frac{\widehat{m}^{\alpha}|\boldsymbol{\zeta}-\hat{\boldsymbol{v}}_{(0)}|^2}{\widehat{T}_{(0)}}-\frac{5}{2}\bigg)\frac{1}{\widehat{T}_{(0)}}\frac{\mathrm{d}\widehat{T}_{(0)}}{\mathrm{d}y}\bigg],\tag{B.1}$$

where $\hat{\boldsymbol{v}}_{(0)}=(0,\hat{v}_{2(0)},0)$. With the following notations

$$\boldsymbol{C}=\big(\boldsymbol{\zeta}-\hat{\boldsymbol{v}}_{(0)}\big)/\sqrt{\widehat{T}_{(0)}},\quad C=|\boldsymbol{C}|,\quad E^{\alpha}(C)=\bigg(\frac{m^{\alpha}}{\pi}\bigg)^{3/2}\exp\big(-\widehat{m}^{\alpha}C^2\big),$$

Eq. (B.1) is transformed into

$$\sum_{\beta=\mathrm{A,B}}K^{\beta\alpha}\chi^{\beta}_{(0)}\mathcal{L}^{\beta\alpha}_{\widehat{T}_{(0)}}\big(\phi^{\beta},\phi^{\alpha}\big)=\frac{C_1}{\hat{n}_{(0)}}\bigg[\frac{1}{\chi^{\alpha}_{(0)}}\frac{\mathrm{d}\chi^{\alpha}_{(0)}}{\mathrm{d}y}+\frac{2\widehat{m}^{\alpha}C_2}{\widehat{T}^{1/2}_{(0)}}\frac{\mathrm{d}\hat{v}_{2(0)}}{\mathrm{d}y}+\bigg(\widehat{m}^{\alpha}|\boldsymbol{C}|^2-\frac{5}{2}\bigg)\frac{1}{\widehat{T}_{(0)}}\frac{\mathrm{d}\widehat{T}_{(0)}}{\mathrm{d}y}\bigg],\tag{B.2}$$

where $\phi^{\alpha}(y,\boldsymbol{C})=f^{\alpha}_{(1)}/f^{\alpha}_{(0)}$ and

$$\mathcal{L}^{\beta\alpha}_{a}(f,g)=\int\big(f'_*-f_*+g'-g\big)E^{\beta}_*b^{\beta\alpha}_{a}(\boldsymbol{e}\cdot\boldsymbol{C}_{\mathrm{rel}}/C_{\mathrm{rel}},C_{\mathrm{rel}})\,\mathrm{d}\Omega(\boldsymbol{e})\,\mathrm{d}\boldsymbol{C}_*,\tag{B.3}$$

where

$$f'_*=f\big(\boldsymbol{C}'_*\big),\quad g'=g\big(\boldsymbol{C}'\big),\quad f_*=f(\boldsymbol{C}_*),\quad g=g(\boldsymbol{C}),\quad E^{\beta}_*=E^{\beta}\big(|\boldsymbol{C}_*|\big),$$
$$\boldsymbol{C}'=\boldsymbol{C}+\frac{\hat{\mu}^{\beta\alpha}}{\widehat{m}^{\alpha}}(\boldsymbol{e}\cdot\boldsymbol{C}_{\mathrm{rel}})\boldsymbol{e},\qquad \boldsymbol{C}'_*=\boldsymbol{C}_*-\frac{\hat{\mu}^{\beta\alpha}}{\widehat{m}^{\beta}}(\boldsymbol{e}\cdot\boldsymbol{C}_{\mathrm{rel}})\,\boldsymbol{e},$$
$$\boldsymbol{C}_{\mathrm{rel}}=\boldsymbol{C}_*-\boldsymbol{C},\qquad C_{\mathrm{rel}}=|\boldsymbol{C}_{\mathrm{rel}}|,$$
$$b^{\beta\alpha}_{a}(\boldsymbol{e}\cdot\boldsymbol{C}_{\mathrm{rel}}/C_{\mathrm{rel}},C_{\mathrm{rel}})=b^{\beta\alpha}(\boldsymbol{e}\cdot\boldsymbol{C}_{\mathrm{rel}}/C_{\mathrm{rel}},\sqrt{a}\,C_{\mathrm{rel}})/\sqrt{a}.$$

The solution ϕ^{α} of (B.2) can be expressed as

$$\phi^{\alpha}=c^{\alpha}_0+\widehat{m}^{\alpha}(\boldsymbol{c}\cdot\boldsymbol{C})+c_4\bigg(\widehat{m}^{\alpha}|\boldsymbol{C}|^2-\frac{3}{2}\bigg)$$
$$-\frac{1}{\hat{n}_{(0)}}\bigg(C_1\mathcal{D}^{\alpha}(|\boldsymbol{C}|)\frac{\mathrm{d}\chi^{\mathrm{A}}_{(0)}}{\mathrm{d}y}+C_1\mathcal{A}^{\alpha}\big(|\boldsymbol{C}|\big)\frac{1}{\widehat{T}_{(0)}}\frac{\mathrm{d}\widehat{T}_{(0)}}{\mathrm{d}y}+C_1C_2\mathcal{B}^{\alpha}\big(|\boldsymbol{C}|\big)\frac{1}{\widehat{T}^{1/2}_{(0)}}\frac{\mathrm{d}\hat{v}_{2(0)}}{\mathrm{d}y}\bigg),$$

where c^{α}_0, $\boldsymbol{c}$, and c_4 are undetermined constants. Here the functions $\mathcal{A}^{\alpha}(|\boldsymbol{C}|)$, $\mathcal{B}^{\alpha}(|\boldsymbol{C}|)$, and $\mathcal{D}^{\alpha}(|\boldsymbol{C}|)$ are the solutions of the following integral equations:

$$\sum_{\beta=\mathrm{A,B}} K^{\beta\alpha}\chi^{\beta}_{(0)}\mathcal{L}^{\beta\alpha}_{\widehat{T}_{(0)}}\left(C_i\mathcal{A}^{\beta}, C_i\mathcal{A}^{\alpha}\right) = -C_i\left(\widehat{m}^{\alpha}|\boldsymbol{C}|^2 - \frac{5}{2}\right), \tag{B.4a}$$

subsidiary condition:

$$\sum_{\alpha=\mathrm{A,B}} \widehat{m}^{\alpha}\chi^{\alpha}_{(0)}\int_0^{\infty} C^4\mathcal{A}^{\alpha}(C)E^{\alpha}(C)\,\mathrm{d}C = 0, \tag{B.4b}$$

$$\sum_{\beta=\mathrm{A,B}} K^{\beta\alpha}\chi^{\beta}_{(0)}\mathcal{L}^{\beta\alpha}_{\widehat{T}_{(0)}}\left(C_{ij}\mathcal{B}^{\beta}, C_{ij}\mathcal{B}^{\alpha}\right) = -2\widehat{m}^{\alpha}C_{ij}, \tag{B.5}$$

$$\sum_{\beta=\mathrm{A,B}} K^{\beta\alpha}\chi^{\beta}_{(0)}\mathcal{L}^{\beta\alpha}_{\widehat{T}_{(0)}}\left(C_i\mathcal{D}^{\beta}, C_i\mathcal{D}^{\alpha}\right) = -\frac{\delta_{\mathrm{A}\alpha} - \delta_{\mathrm{B}\alpha}}{\chi^{\alpha}_{(0)}}C_i, \tag{B.6a}$$

subsidiary condition:

$$\sum_{\beta=\mathrm{A,B}} \widehat{m}^{\alpha}\chi^{\alpha}_{(0)}\int_0^{\infty} C^4\mathcal{D}^{\alpha}(C)E^{\alpha}(C)\,\mathrm{d}C = 0, \tag{B.6b}$$

where $C_{ij} = C_iC_j - \frac{1}{3}|\boldsymbol{C}|\delta_{ij}$, $\delta_{\mathrm{AA}} = \delta_{\mathrm{BB}} = 1$, and $\delta_{\mathrm{AB}} = \delta_{\mathrm{BA}} = 0$. The functions are orthogonal to the collision invariants. The undetermined constants c^{α}_0, $\boldsymbol{c}$, and c_4 can be expressed by the first few moments of $f^{\alpha}_{(1)}$, and the following expression is finally obtained:

$$\begin{aligned} f^{\alpha}_{(1)} = \frac{\hat{n}^{\alpha}_{(0)}}{\widehat{T}^{3/2}_{(0)}}E^{\alpha}\left(|\boldsymbol{C}|\right)\Bigg[&\frac{\hat{p}^{\alpha}_{(1)}}{\hat{p}^{\alpha}_{(0)}} + \frac{2\widehat{m}^{\alpha}}{\sqrt{\widehat{T}_{(0)}}}\left(\hat{v}_{1(1)}C_1 + \hat{v}_{2(1)}C_2\right) + \frac{\widehat{T}_{(1)}}{\widehat{T}_{(0)}}\left(\widehat{m}^{\alpha}|\boldsymbol{C}|^2 - \frac{5}{2}\right) \\ &- \frac{1}{\hat{n}_{(0)}}\left(C_1\mathcal{D}^{\alpha}\left(|\boldsymbol{C}|\right)\frac{\mathrm{d}\chi^{\mathrm{A}}_{(0)}}{\mathrm{d}y} + C_1\mathcal{A}^{\alpha}\left(|\boldsymbol{C}|\right)\frac{1}{\widehat{T}_{(0)}}\frac{\mathrm{d}\widehat{T}_{(0)}}{\mathrm{d}y} + C_1C_2\mathcal{B}^{\alpha}(|\boldsymbol{C}|)\frac{1}{\widehat{T}^{1/2}_{(0)}}\frac{\mathrm{d}\hat{v}_{2(0)}}{\mathrm{d}y}\right)\Bigg]. \end{aligned} \tag{B.7}$$

In the derivation, it is clarified that the temperature is common to species also at $\mathrm{O}(\epsilon^1)$:

$$\widehat{T}^{\mathrm{A}}_{(1)} = \widehat{T}^{\mathrm{B}}_{(1)} = \widehat{T}_{(1)}.$$

B.2. Symmetry property of $\mathcal{L}^{\beta\alpha}_a$ and related results

Thanks to the symmetry property of $\mathcal{L}^{\beta\alpha}_a$, the following equality holds:

$$\begin{aligned} &\int \psi(\boldsymbol{C})\mathcal{L}^{\beta\alpha}_a(f,g)E^{\alpha}\left(|\boldsymbol{C}|\right)\mathrm{d}\boldsymbol{C} + \int \phi(\boldsymbol{C})\mathcal{L}^{\alpha\beta}_a(g,f)E^{\beta}\left(|\boldsymbol{C}|\right)\mathrm{d}\boldsymbol{C} \\ &\quad = -\frac{1}{2}\int \left(\phi'_* - \phi_* + \psi' - \psi\right)\left(f'_* - f_* + g' - g\right)E^{\alpha}E^{\beta}_*b^{\beta\alpha}_a(\boldsymbol{e}\cdot\boldsymbol{C}_{\mathrm{rel}}/C_{\mathrm{rel}}, C_{\mathrm{rel}})\,\mathrm{d}\Omega(\boldsymbol{e})\,\mathrm{d}\boldsymbol{C}_*\,\mathrm{d}\boldsymbol{C}, \end{aligned} \tag{B.8}$$

where

$$\begin{aligned} &f'_* = f\left(\boldsymbol{C}'_*\right), \quad g' = g\left(\boldsymbol{C}'\right), \quad f_* = f(\boldsymbol{C}_*), \quad g = g(\boldsymbol{C}), \\ &\phi'_* = \phi\left(\boldsymbol{C}'_*\right), \quad \psi' = \psi\left(\boldsymbol{C}'\right), \quad \phi_* = \phi(\boldsymbol{C}_*), \quad \psi = \psi(\boldsymbol{C}), \quad E^{\beta}_* = E^{\beta}\left(|\boldsymbol{C}_*|\right), \\ &\boldsymbol{C}' = \boldsymbol{C} + \frac{\hat{\mu}^{\beta\alpha}}{\widehat{m}^{\alpha}}(\boldsymbol{e}\cdot\boldsymbol{C}_{\mathrm{rel}})\boldsymbol{e}, \qquad \boldsymbol{C}'_* = \boldsymbol{C}_* - \frac{\hat{\mu}^{\beta\alpha}}{\widehat{m}^{\beta}}(\boldsymbol{e}\cdot\boldsymbol{C}_{\mathrm{rel}})\,\boldsymbol{e}, \\ &\boldsymbol{C}_{\mathrm{rel}} = \boldsymbol{C}_* - \boldsymbol{C}, \qquad C_{\mathrm{rel}} = |\boldsymbol{C}_{\mathrm{rel}}|. \end{aligned}$$

The proof is classical and is omitted here. From this equality one can derive the following:

(1) for arbitrary functions f^α and g^α and for arbitrary constants $C^{\beta\alpha}$ such that $C^{\beta\alpha} = C^{\alpha\beta}$,

$$\sum_{\alpha=\mathrm{A,B}}\sum_{\beta=\mathrm{A,B}} C^{\beta\alpha}\int g^\alpha(\boldsymbol{C})\mathcal{L}_a^{\beta\alpha}\big(f^\beta, f^\alpha\big)E^\alpha\big(|\boldsymbol{C}|\big)\,\mathrm{d}\boldsymbol{C} = \sum_{\alpha=\mathrm{A,B}}\sum_{\beta=\mathrm{A,B}} C^{\beta\alpha}\int f^\alpha(\boldsymbol{C})\mathcal{L}_a^{\beta\alpha}\big(g^\beta, g^\alpha\big)E^\alpha\big(|\boldsymbol{C}|\big)\,\mathrm{d}\boldsymbol{C}; \tag{B.9}$$

(2) for an arbitrary function f^α and for arbitrary constants $C^{\beta\alpha}$ such that $C^{\beta\alpha} = C^{\alpha\beta}$,

$$\sum_{\alpha=\mathrm{A,B}}\sum_{\beta=\mathrm{A,B}} C^{\beta\alpha}\int f^\alpha(\boldsymbol{C})\mathcal{L}_a^{\beta\alpha}\big(f^\beta, f^\alpha\big)E^\alpha\big(|\boldsymbol{C}|\big)\,\mathrm{d}\boldsymbol{C} \leqslant 0. \tag{B.10}$$

The equality in (B.10) holds if and only if f^α is the collision invariant:

$$f^\alpha = c_0^\alpha + \widehat{m}^\alpha(\boldsymbol{c}\cdot\boldsymbol{C}) + \widehat{m}^\alpha c_4|\boldsymbol{C}|^2,$$

where c_0^α, $\boldsymbol{c}$, and c_4 are arbitrary constants.

Eqs. (B.9) and (B.10) are used in obtaining important properties of the moments of $\mathcal{A}^\alpha$, $\mathcal{B}^\alpha$, and $\mathcal{D}^\alpha$ listed below:

(1) Putting $g^\alpha = C_i\mathcal{D}^\alpha$ and $f^\alpha = C_i\mathcal{A}^\alpha$ in (B.9) with $C^{\beta\alpha} = K^{\beta\alpha}\chi_{(0)}^\beta\chi_{(0)}^\alpha$ and $a = \widehat{T}_{(0)}$ yields

$$\sum_{\alpha=\mathrm{A,B}}\chi_{(0)}^\alpha\int_0^\infty C^4\left(\widehat{m}^\alpha C^2 - \frac{5}{2}\right)\mathcal{D}^\alpha E^\alpha\,\mathrm{d}C = \int_0^\infty C^4\big(\mathcal{A}^\mathrm{A}E^\mathrm{A} - \mathcal{A}^\mathrm{B}E^\mathrm{B}\big)\,\mathrm{d}C. \tag{B.11}$$

(2) Putting $f^\alpha = C_i(k_1\mathcal{A}^\alpha + k_2\mathcal{D}^\alpha)$ in (B.10) with $C^{\beta\alpha} = K^{\beta\alpha}\chi_{(0)}^\beta\chi_{(0)}^\alpha$ and $a = \widehat{T}_{(0)}$ yields

$$-k_1^2\sum_{\alpha=\mathrm{A,B}}\chi_{(0)}^\alpha\int_0^\infty C^4\left(\widehat{m}^\alpha C^2 - \frac{5}{2}\right)\mathcal{A}^\alpha E^\alpha\,\mathrm{d}C - 2k_1k_2\int_0^\infty C^4\big(\mathcal{A}^\mathrm{A}E^\mathrm{A} - \mathcal{A}^\mathrm{B}E^\mathrm{B}\big)\,\mathrm{d}C - k_2^2\int_0^\infty C^4\big(\mathcal{D}^\mathrm{A}E^\mathrm{A} - \mathcal{D}^\mathrm{B}E^\mathrm{B}\big)\,\mathrm{d}C \leqslant 0. \tag{B.12}$$

This leads to

(a) with $k_1 = 0$ and $k_2 = 1$,

$$\int_0^\infty C^4\big(\mathcal{D}^\mathrm{A}E^\mathrm{A} - \mathcal{D}^\mathrm{B}E^\mathrm{B}\big)\,\mathrm{d}C > 0, \tag{B.13}$$

(b) with $k_1 = 1$ and $k_2 = -\int_0^\infty C^4(\mathcal{A}^\mathrm{A}E^\mathrm{A} - \mathcal{A}^\mathrm{B}E^\mathrm{B})\,\mathrm{d}C/\int_0^\infty C^4(\mathcal{D}^\mathrm{A}E^\mathrm{A} - \mathcal{D}^\mathrm{B}E^\mathrm{B})\,\mathrm{d}C$,

$$\sum_{\alpha=\mathrm{A,B}}\chi_{(0)}^\alpha\int_0^\infty C^4\left(\widehat{m}^\alpha C^2 - \frac{5}{2}\right)\mathcal{A}^\alpha E^\alpha\,\mathrm{d}C - \frac{(\int_0^\infty C^4(\mathcal{A}^\mathrm{A}E^\mathrm{A} - \mathcal{A}^\mathrm{B}E^\mathrm{B})\,\mathrm{d}C)^2}{\int_0^\infty C^4(\mathcal{D}^\mathrm{A}E^\mathrm{A} - \mathcal{D}^\mathrm{B}E^\mathrm{B})\,\mathrm{d}C} > 0. \tag{B.14}$$

Here, in (B.13) and (B.14), the equality drops because $k_1\mathcal{A}^\alpha + k_2\mathcal{D}^\alpha$ is a non-zero function orthogonal to the collision invariants.

(3) Putting $f^\alpha = C_1C_2\mathcal{B}^\alpha$ in (B.10) with $C^{\beta\alpha} = K^{\beta\alpha}\chi_{(0)}^\beta\chi_{(0)}^\alpha$ and $a = \widehat{T}_{(0)}$ yields

$$\sum_{\alpha=\mathrm{A,B}}\widehat{m}^\alpha\chi_{(0)}^\alpha\int_0^\infty C^6\mathcal{B}^\alpha E^\alpha\,\mathrm{d}C > 0. \tag{B.15}$$

Here, the equality drops because $\mathcal{B}^\alpha$ is a non-zero function orthogonal to the collision invariants.

Eq. (B.12) leads to the different expressions for $\widehat{D}_\mathrm{T}$ in (27b). Eqs. (B.13)–(B.15) lead to the positivity of $\widehat{D}_\mathrm{AB}$, $\hat{\lambda}$, and $\hat{\mu}$ [see (27a), (27e), and (27d)].

Appendix C. Derivation of the equations for $\chi^\mathrm{A}_{(1)}$, $\hat{v}_{2(1)}$, and $\widehat{T}_{(1)}$ in the case of evaporation

In this appendix, we will derive the equations for $\chi^\mathrm{A}_{(1)}$, $\hat{v}_{2(1)}$, and $\widehat{T}_{(1)}$ under the assumption that $f^\alpha_{(0)}$ is uniform with respect to y. This assumption is true in the case of evaporation.

Consider the Maxwellian M^α with the number density $\hat{n}^\alpha$, temperature $\widehat{T}$, and flow velocity $\hat{\boldsymbol{v}} = (\hat{v}_1, \hat{v}_2, 0)$ that are the same as those of f^α:

$$M^\alpha = \frac{\hat{n}^\alpha}{\widehat{T}^{3/2}}\left(\frac{\widehat{m}^\alpha}{\pi}\right)^{3/2}\exp\left(-\frac{\widehat{m}^\alpha|\boldsymbol{\zeta} - \hat{\boldsymbol{v}}|^2}{\widehat{T}}\right).$$

If we expand it in a power series of ϵ:

$$M^\alpha = M^\alpha_{(0)} + M^\alpha_{(1)}\epsilon + \cdots,$$

the component functions of the expansion satisfy the integral equations

$$\sum_{\beta=\mathrm{A,B}} K^{\beta\alpha}\hat{J}^{\beta\alpha}\big(M^\beta_{(0)}, M^\alpha_{(0)}\big) = 0, \tag{C.1}$$

$$\sum_{\beta=\mathrm{A,B}} K^{\beta\alpha}\big[\hat{J}^{\beta\alpha}\big(M^\beta_{(1)}, M^\alpha_{(0)}\big) + \hat{J}^{\beta\alpha}\big(M^\beta_{(0)}, M^\alpha_{(1)}\big)\big] = 0, \tag{C.2}$$

$$\sum_{\beta=\mathrm{A,B}} K^{\beta\alpha}\big[\hat{J}^{\beta\alpha}\big(M^\beta_{(2)}, M^\alpha_{(0)}\big) + \hat{J}^{\beta\alpha}\big(M^\beta_{(0)}, M^\alpha_{(2)}\big) + \hat{J}^{\beta\alpha}\big(M^\beta_{(1)}, M^\alpha_{(1)}\big)\big] = 0, \tag{C.3}$$

and so on because

$$\sum_{\beta=\mathrm{A,B}} K^{\beta\alpha}\hat{J}^{\beta\alpha}\big(M^\beta, M^\alpha\big) = 0.$$

On the other hand, if $f^\alpha_{(0)}$ is uniform with respect to y, f^α is a local equilibrium distribution up to the order of ϵ [see (B.7)], i.e.,

$$f^\alpha_{(0)} = M^\alpha_{(0)} = \frac{\hat{n}^\alpha_{(0)}}{\widehat{T}^{3/2}_{(0)}}E^\alpha\big(|\boldsymbol{C}|\big),$$

$$f^\alpha_{(1)} = M^\alpha_{(1)} = \frac{\hat{n}^\alpha_{(0)}}{\widehat{T}^{3/2}_{(0)}}E^\alpha\big(|\boldsymbol{C}|\big)\left[\frac{\hat{p}^\alpha_{(1)}}{\hat{p}^\alpha_{(0)}} + \frac{2\widehat{m}^\alpha}{\sqrt{\widehat{T}_{(0)}}}\big(\hat{v}_{1(1)}C_1 + \hat{v}_{2(1)}C_2\big) + \frac{\widehat{T}_{(1)}}{\widehat{T}_{(0)}}\left(\widehat{m}^\alpha|\boldsymbol{C}|^2 - \frac{5}{2}\right)\right].$$

Consequently, in addition to $\widehat{T}^\alpha_{(1)} = \widehat{T}_{(1)}$, the following relations hold

$$\hat{v}^\alpha_{1(1)} = \hat{v}_{1(1)} = \widehat{w}_{1(1)}, \qquad \hat{v}^\alpha_{2(1)} = \hat{v}_{2(1)} = \widehat{w}_{2(1)},$$
$$\hat{p}^\alpha_{ij(1)} = \hat{p}^\alpha_{(1)}\delta_{ij}, \quad \hat{p}_{ij(1)} = \hat{p}_{(1)}\delta_{ij}, \quad \hat{q}^\alpha_{i(1)} = \hat{q}_{i(1)} = 0.$$

Now keeping in mind the properties summarized above, we consider the integral equation for $f^\alpha_{(2)}$:

$$\sum_{\beta=\mathrm{A,B}} K^{\beta\alpha}\big[\hat{J}^{\beta\alpha}\big(f^\beta_{(2)}, f^\alpha_{(0)}\big) + \hat{J}^{\beta\alpha}\big(f^\beta_{(0)}, f^\alpha_{(2)}\big)\big] = \zeta_1\frac{\partial f^\alpha_{(1)}}{\partial y} - \sum_{\beta=\mathrm{A,B}} K^{\beta\alpha}\hat{J}^{\beta\alpha}\big(f^\beta_{(1)}, f^\alpha_{(1)}\big).$$

Subtraction of (C.3) yields

$$\sum_{\beta=\mathrm{A,B}} K^{\beta\alpha}\big[\hat{J}^{\beta\alpha}\big(f^\beta_{(2)} - M^\beta_{(2)}, f^\alpha_{(0)}\big) + \hat{J}^{\beta\alpha}\big(f^\beta_{(0)}, f^\alpha_{(2)} - M^\alpha_{(2)}\big)\big] = \zeta_1\frac{\partial f^\alpha_{(1)}}{\partial y},$$

leading to

$$\sum_{\beta=\mathrm{A,B}} K^{\beta\alpha}\chi_{(0)}^{\beta}\mathcal{L}_{\widehat{T}_{(0)}}^{\beta\alpha}(\psi^\beta,\psi^\alpha)=\frac{C_1}{\hat{n}_{(0)}}\left[\frac{1}{\chi_{(0)}^{\alpha}}\frac{\mathrm{d}\chi_{(1)}^{\alpha}}{\mathrm{d}y}+\frac{2\widehat{m}^{\alpha}C_2}{\widehat{T}_{(0)}^{1/2}}\frac{\mathrm{d}\hat{v}_{2(1)}}{\mathrm{d}y}+\frac{1}{\widehat{T}_{(0)}}\left(\widehat{m}^{\alpha}|\boldsymbol{C}|^2-\frac{5}{2}\right)\frac{\mathrm{d}\widehat{T}_{(1)}}{\mathrm{d}y}\right],$$

where $\psi^\alpha=(f_{(2)}^{\alpha}-M_{(2)}^{\alpha})/f_{(0)}^{\alpha}$. Here $\hat{p}_{(1)}=\text{const}$ and $\hat{v}_{1(1)}=\text{const}$, which are from (44) and (49), are used. Note that the quantities with subscript (0) are all constants. Hence, in the same way as in Appendix B.1, we obtain

$$\begin{aligned}f_{(2)}^{\alpha}={}&M_{(2)}^{\alpha}+\frac{\hat{n}_{(0)}^{\alpha}}{\widehat{T}_{(0)}^{3/2}}E^{\alpha}\big(|\boldsymbol{C}|\big)\bigg[c_0^{\alpha}+\widehat{m}^{\alpha}(\boldsymbol{c}\cdot\boldsymbol{C})+c_4\left(\widehat{m}^{\alpha}|\boldsymbol{C}|^2-\frac{3}{2}\right)\\&-\frac{1}{\hat{n}_{(0)}}\left(C_1\mathcal{D}^{\alpha}\big(|\boldsymbol{C}|\big)\frac{\mathrm{d}\chi_{(1)}^{\mathrm{A}}}{\mathrm{d}y}+C_1\mathcal{A}^{\alpha}\big(|\boldsymbol{C}|\big)\frac{1}{\widehat{T}_{(0)}}\frac{\mathrm{d}\widehat{T}_{(1)}}{\mathrm{d}y}+C_1C_2\mathcal{B}^{\alpha}\big(|\boldsymbol{C}|\big)\frac{1}{\widehat{T}_{(0)}^{1/2}}\frac{\mathrm{d}\hat{v}_{2(1)}}{\mathrm{d}y}\right)\bigg],\end{aligned}$$

where c_0^{α}, $\boldsymbol{c}$, and c_4 are again undetermined constants. They are determined by the first few moments of $f_{(2)}^{\alpha}$, and finally the following expression is obtained:

$$f_{(2)}^{\alpha}=M_{(2)}^{\alpha}-\frac{\chi_{(0)}^{\alpha}}{\widehat{T}_{(0)}^{3/2}}E^{\alpha}\big(|\boldsymbol{C}|\big)\left(C_1\mathcal{D}^{\alpha}\big(|\boldsymbol{C}|\big)\frac{\mathrm{d}\chi_{(1)}^{\mathrm{A}}}{\mathrm{d}y}+C_1\mathcal{A}^{\alpha}\big(|\boldsymbol{C}|\big)\frac{1}{\widehat{T}_{(0)}}\frac{\mathrm{d}\widehat{T}_{(1)}}{\mathrm{d}y}+C_1C_2\mathcal{B}^{\alpha}\big(|\boldsymbol{C}|\big)\frac{1}{\widehat{T}_{(0)}^{1/2}}\frac{\mathrm{d}\hat{v}_{2(1)}}{\mathrm{d}y}\right). \quad \text{(C.4)}$$

This result leads to

(1) $$\hat{v}_{1(2)}^{\mathrm{A}}-\hat{v}_{1(2)}^{\mathrm{B}}=-\frac{\widehat{D}_{\mathrm{AB}}}{\chi_{(0)}^{\mathrm{A}}\chi_{(0)}^{\mathrm{B}}}\frac{\widehat{T}_{(0)}^{1/2}}{\hat{n}_{(0)}}\left(\frac{\mathrm{d}\chi_{(1)}^{\mathrm{A}}}{\mathrm{d}y}+\frac{k_{\mathrm{T}}}{\widehat{T}_{(0)}}\frac{\mathrm{d}\widehat{T}_{(1)}}{\mathrm{d}y}\right),$$

or, equivalently,

$$\hat{v}_{1(2)}^{\mathrm{A}}=\widehat{w}_{1(2)}-\frac{\widehat{D}_{\mathrm{AB}}}{\chi_{(0)}^{\mathrm{A}}}\frac{\widehat{T}_{(0)}^{1/2}}{\hat{n}_{(0)}}\left(\frac{\mathrm{d}\chi_{(1)}^{\mathrm{A}}}{\mathrm{d}y}+\frac{k_{\mathrm{T}}}{\widehat{T}_{(0)}}\frac{\mathrm{d}\widehat{T}_{(1)}}{\mathrm{d}y}\right), \quad \text{(C.5)}$$

(2) $$\sum_{\alpha=\mathrm{A,B}}\int\widehat{m}^{\alpha}\zeta_1\zeta_2 f_{(2)}^{\alpha}\,\mathrm{d}\boldsymbol{\zeta}=\frac{1}{2}\hat{p}_{12(2)}+\hat{\rho}_{(0)}\hat{v}_{1(1)}\hat{v}_{2(1)}+\big(\hat{\rho}_{(0)}\hat{v}_{1(2)}+\hat{\rho}_{(1)}\hat{v}_{1(1)}\big)\hat{v}_{2(0)},$$

and

$$\hat{p}_{12(2)}=-\hat{\mu}\widehat{T}_{(0)}^{1/2}\frac{\mathrm{d}\hat{v}_{2(1)}}{\mathrm{d}y},$$

(3) $$\begin{aligned}\sum_{\alpha=\mathrm{A,B}}\int\widehat{m}^{\alpha}|\boldsymbol{\zeta}|^2\zeta_1 f_{(2)}^{\alpha}\,\mathrm{d}\boldsymbol{\zeta}={}&-\hat{\lambda}\widehat{T}_{(0)}^{1/2}\frac{\mathrm{d}\widehat{T}_{(1)}}{\mathrm{d}y}+\frac{5}{2}\big(\hat{p}_{(1)}\widehat{w}_{1(1)}+\hat{p}_{(0)}\widehat{w}_{1(2)}\big)+k_{\mathrm{T}}\hat{p}_{(0)}\big(\hat{v}_{1(2)}^{\mathrm{A}}-\hat{v}_{1(2)}^{\mathrm{B}}\big)\\&+\hat{p}_{12(2)}\hat{v}_{2(0)}+2\hat{\rho}_{(0)}\hat{v}_{1(1)}\hat{v}_{2(0)}\hat{v}_{2(1)}+\big(\hat{\rho}_{(0)}\hat{v}_{1(2)}+\hat{\rho}_{(1)}\hat{v}_{1(1)}\big)\hat{v}_{2(0)}^2,\end{aligned}$$

where $\widehat{w}_{1(2)}=\chi_{(0)}^{\mathrm{A}}\hat{v}_{1(2)}^{\mathrm{A}}+\chi_{(0)}^{\mathrm{B}}\hat{v}_{1(2)}^{\mathrm{B}}$ because $\hat{v}_{1(1)}^{\alpha}=\hat{v}_{1(1)}=\widehat{w}_{1(1)}$.

With these expressions, it is easy to derive the equations for $\chi_{(1)}^{\mathrm{A}}$, $\hat{v}_{2(1)}$, and $\widehat{T}_{(1)}$. First, (22a) with $m=3$ is equivalent to

$$\frac{\mathrm{d}}{\mathrm{d}y}\big(\hat{n}_{(0)}^{\mathrm{A}}\hat{v}_{1(2)}^{\mathrm{A}}+\hat{n}_{(1)}^{\mathrm{A}}\hat{v}_{1(1)}\big)=0,$$

$$\frac{\mathrm{d}}{\mathrm{d}y}\big(\hat{n}_{(0)}\widehat{w}_{1(2)}+\hat{n}_{(1)}\widehat{w}_{1(1)}\big)=0 \quad \left(\text{or } \frac{\mathrm{d}}{\mathrm{d}y}(\hat{\rho}_{(0)}\hat{v}_{1(2)}+\hat{\rho}_{(1)}\hat{v}_{1(1)})=0\right).$$

Substitution of (C.5) into the first equation and using the second equation yields

$$\frac{\mathrm{d}}{\mathrm{d}y}\left[\widehat{D}_{\mathrm{AB}}\widehat{T}_{(0)}^{1/2}\left(\frac{\mathrm{d}\chi_{(1)}^{\mathrm{A}}}{\mathrm{d}y}+\frac{k_{\mathrm{T}}}{\widehat{T}_{(0)}}\frac{\mathrm{d}\widehat{T}_{(1)}}{\mathrm{d}y}\right)\right]=\hat{n}_{(0)}\widehat{w}_{1(1)}\frac{\mathrm{d}\chi_{(1)}^{\mathrm{A}}}{\mathrm{d}y}.$$

Next, the first line of (22b) with $i = 2$ and $m = 3$ yields

$$\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}y}\left(\hat{\mu}\widehat{T}_{(0)}^{1/2}\frac{\mathrm{d}\hat{v}_{2(1)}}{\mathrm{d}y}\right) = \hat{\rho}_{(0)}\hat{v}_{1(1)}\frac{\mathrm{d}\hat{v}_{2(1)}}{\mathrm{d}y}.$$

Finally, the second line of (22b) with $m = 3$ yields

$$\frac{\mathrm{d}}{\mathrm{d}y}\left(\hat{\lambda}\widehat{T}_{(0)}^{1/2}\frac{\mathrm{d}\widehat{T}_{(1)}}{\mathrm{d}y} - k_{\mathrm{T}}\hat{p}_{(0)}\left(\hat{v}_{1(2)}^{\mathrm{A}} - \hat{v}_{1(2)}^{\mathrm{B}}\right)\right) = \frac{5}{2}\hat{n}_{(0)}\widehat{w}_{1(1)}\frac{\mathrm{d}\widehat{T}_{(1)}}{\mathrm{d}y} + \frac{\mathrm{d}}{\mathrm{d}y}\left(2\hat{\rho}_{(0)}\hat{v}_{1(1)}\hat{v}_{2(0)}\hat{v}_{2(1)} - \hat{\mu}\widehat{T}_{(0)}^{1/2}\hat{v}_{2(0)}\frac{\mathrm{d}\hat{v}_{2(1)}}{\mathrm{d}y}\right).$$

These are the desired equations.

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